Subject: Business Economics

Course: B.A., 6th Semester, Undergraduate.

Paper No: 631

Paper Title: Advance Mathematical Techniques.

Unit No.: 3 (Three)

Title: Linear Programming.

Lecture No: 1 (One)

Title: Duality in Linear Programming.

FAQs

Q-1. What is dual?

Ans. Dual is a twin linear programming problem.

Q-2. What do you mean by primal and dual problem?

Ans. Every linear programming problem (LPP) has another linear programming problem associated with it. The original problem is called the "Primal" while the other is called its "Dual".

Q-3 State the general rules for writing the dual of the primal.

Ans.	Primal Problem	Dual Problem
1	Objective function is maximization	Objective function is minimization
2	Requirement vector	Price vector
3	Coefficient matrix A	Transpose of Coefficient matrix A^{T}
4	Constraints with ≤ sign	Constraints with ≥ sign
5	Relation	Variable
6	<i>ith</i> —inequality	i^{th} -variable $W_i \ge 0$
7	<i>ith</i> –constraint an equality	i^{th} -variable unrestricted in sign
8	Variable	Relation
9	If i^{th} –variable $X_i > 0$	<i>ith</i> –relation strictly equality
10	If i^{th} –variable unrestricted in sign	i^{th} –constraint strict equality
11	If i^{th} —slack variable positive	<i>ith</i> –variable is zero
12	If i^{th} –variable is zero	i^{th} – surplus variable positive
13	Finite optimal solution	Finite optimal solution equal to that of the
		primal
14	Unbounded solution	Infeasible solution

Q-4. What is dual of the dual?

Ans. Dual of dual is primal.

Q-5. What is the relationship between the feasible solutions of the primal and its dual?

- Ans.
 - (i) If either the primal or the dual problem has a finite optimal solution, then the dual problem also has a finite optimal solution. Furthermore, the optimal values of the objective functions of both the problems are same.
 - (ii) If either problem has an unbounded solution, then the other problem has infeasible solution.
 - (iii) Both problems may not have any solution.

Q-5. State the Basic Duality Theorem.

Ans. If $X_0(W_0)$ is an optimal solution to the primal (dual), then there exists a feasible solution $W_0(X_0)$ to the dual (primal), such that $CX_0 = b^T W_0$. This is known as Basic Duality Theorem.

Q-6 What can you say about the solution of the dual problem if the primal has an unbounded solution?

Ans. If dual problem has an unbounded solution, then the primal problem has infeasible solution.

Q-7 State the Fundamental Duality Theorem.

Ans. Fundamental Duality Theorem

- i. If either the primal or the dual problem has a finite optimal solution, then the dual problem also has a finite optimal solution. Furthermore, the optimal values of the objective functions of both the problems are same.
- ii. If either problem has an unbounded solution, then the other problem has infeasible solution.
- iii. Both problems may not have any solution.

Q-8 What is the nature of the dual variable if the constraint of the primal problem is an equation?

Ans. The dual variable will be unrestricted in sign.

Q-9 State Complementary-Slackness Theorem.

Ans.

For the optimal feasible solutions of the primal and dual problems

- i. if the inequality occurs in the i^{th} relation of either system, the corresponding slack or surplus variable S_{n+i} is positive, then the i^{th} variable W_i of its dual is zero.
- ii. if the j^{th} variable is positive for either system, the j^{th} dual constraint holds as a strict equality. That is, the corresponding slack/surplus variable W_{m+j} is zero.