[Frequently Asked Questions]

[Constrained Optimization]

Subject:	Business Economics
Course:	B. A. (Hons.), 6th Semester, Undergraduate
Paper No. & Title:	Paper – 631 Advanced Mathematical Techniques
Unit No. & Title:	Unit – 2 Function of Two Variables
Lecture No. & Title:	4 (Four) Constrained Optimization

Frequently Asked Questions

1. What is non-linear programming problem?

- Ans. A problem is said to be non-linear if either objective function or constraints or both are non-linear.

2. Discuss the notion of the point of maxima / minima.

- Ans. A function $f(x)$ has *maxima* at a point x_0 if for arbitrary small $|h|$, $f(x_0+h)-f(x_0)<0$. A function $f(x)$ has *minima* at a point x_0 if for arbitrary small $|h|$, $f(x_0+h)-f(x_0)>0$.

3. What do you mean by point of inflexion?

- Ans. If the Hessian matrix H is indefinite at the point X_0, then the point X_0 is called point of inflexion.

4. State Lagrangian Multiplier method for one equality constraint.

- Ans. Consider the non-linear programming problem involving $n-$ variables and single constraint as

Optimize $\quad z=f(X), X\in \mathrm{R}^n$

subject to the constraint

$$g(X)=0; x\geq 0$$

The Lagrangian function is

$$L(X,\lambda)=f(X)-\lambda g(X)$$

The necessary conditions for an extreme point are

$$\frac{\partial L}{\partial x_j}=\frac{\partial f}{\partial x_j}-\lambda\frac{\partial g}{\partial x_j}=0; j=1,2,\ldots,n$$

$$\frac{\partial L}{\partial \lambda}=-g(X)=0$$

Then $\lambda=\frac{\partial f}{\partial x_j}\Big/\frac{\partial g}{\partial x_j}; j=1,2,\ldots,n$.

5. When one needs to use Kuhn-Tucker conditions?

- Ans. When constraints are of inequality type in non-linear programming problem, we use Kuhn-Tucker conditions.

6. State Kuhn-Tucker conditions.

- Ans. Kuhn-Tucker conditions for NLPP

Maximize $\quad z=f(X), X\in \mathbb{R}^n$

subject to the constraint

$$g_i(X)\leq b_i; i=1,2,\ldots,m$$

$$x_j\geq 0; j=1,2,\ldots,n$$

The Lagrangian function is

$$L(X,\lambda) = f(X) - \lambda g(X)$$

The necessary conditions for an extreme point are

(i) $$\frac{\partial L}{\partial x_j} = \frac{\partial f}{\partial x_j} - \lambda \frac{\partial g}{\partial x_j} = 0; j = 1,2,\ldots,n$$

(ii) $\lambda_i \left(g_i(X) - b_i\right) = 0$

(iii) $g_i(X) \leq b_i$

(iv) $\lambda_i \geq 0; i = 1,2,\ldots,m$

7. What is the nature of the Lagrange's multiplier when the objective function is to be maximized?

- Ans. For maximization problem, the Lagrange's multiplier is non-negative.

8. When constraints are of equality type, what can you say about the signs of Lagrangian's multiplier?

- Ans. When constraints are of equality type, the signs of Lagrangian's multiplier are unrestricted in nature.