[Glossary]

[Concept of Convexity/Concavity]

Subject:	Business Economics
Course:	B. A. (Hons.), 6th Semester, Undergraduate
Paper No. & Title:	Paper – 631 Advanced Mathematical Techniques
Unit No. & Title:	Unit – 2 Function of Two Variables
Lecture No. & Title:	Lecture – 3 Concept of Convexity/ Concavity

Glossary

- A set $X \subset \mathbb{R}^n$ is ***convex*** if for every pair of points x_1 and $x_2 \in X$ and any $\lambda \in [0, 1]$, the point $x = \lambda x_1 + (1-\lambda)x_2$ also belongs to the set X.
- A set $X \subset \mathbb{R}^n$ is ***strictly convex***, if for every pair of points x_1 and $x_2 \in X$ and any $\lambda \in (0, 1)$, x is an interior point of X where $x = \lambda x_1 + (1-\lambda)x_2$.
- The function f is ***convex*** if $f(x) \leq \lambda f(x_1) + (1-\lambda) f(x_2)$ where $x = \lambda x_1 + (1-\lambda) x_2$ and $\lambda \in [0, 1]$.
- The function f is ***concave*** if $f(x) \geq \lambda f(x_1) + (1-\lambda) f(x_2)$ where $x = \lambda x_1 + (1-\lambda) x_2$ and $\lambda \in [0, 1]$.
- A ***level set*** of the function $y = f(x_1, x_2, \ldots, x_n)$ is the set

$$L = \{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n : f(x_1, x_2, \ldots, x_n) = c\}$$

  for some real number c.
- The ***better set*** of a point $(x_{10}, x_{20}, \ldots, x_{n0})$ is

  $B(x_{10}, x_{20}, \ldots, x_{n0}) = \{(x_{10}, x_{20}, \ldots, x_{n0}) : f(x_1, x_2, \ldots, x_n) \geq f(x_{10}, x_{20}, \ldots, x_{n0})\}$.
- A function f with domain $X \subset \mathbb{R}^n$ is ***quasiconcave function***, if every point in X, the better set B of that point is a convex set.
- The ***worse set*** of a point $(x_{10}, x_{20}, \ldots, x_{n0})$ is

  $W(x_{10}, x_{20}, \ldots, x_{n0}) = \{(x_{10}, x_{20}, \ldots, x_{n0}) : f(x_1, x_2, \ldots, x_n) \leq f(x_{10}, x_{20}, \ldots, x_{n0})\}$.
- A function $f(x_1, x_2, \ldots, x_n)$ with domain $X \subset \mathbb{R}^n$ is ***quasiconvex*** if every $(x_{10}, x_{20}, \ldots, x_{n0}) \in X$, the worse $W(x_{10}, x_{20}, \ldots, x_{n0})$ is a convex set.
- If the function $z = f(x,y)$ defined on $\mathbb{R}^2$ is twice continuously differentiable and $d^2z = f_{xx}dx^2 + 2f_{xy}dxdy + f_{yy}dy^2 > 0$ whenever at least one of the dx or dy is non-zero, then $z = f(x,y)$ is a strictly convex function.
- If the function $z = f(x,y)$ defined on $\mathbb{R}^2$ is twice continuously differentiable and $d^2z = f_{xx}dx^2 + 2f_{xy}dxdy + f_{yy}dy^2 < 0$ whenever at least one of the dx or dy is non-zero, then $z = f(x,y)$ is a strictly concave function.
- If the function $z = f(x,y)$ defined on $\mathbb{R}^2$ is twice continuously differentiable then it is convex if and only if

$$d^2z = f_{xx}dx^2 + 2f_{xy}dxdy + f_{yy}dy^2 \geq 0.$$

- If the function $z = f(x,y)$ defined on $\mathbb{R}^2$ is twice continuously differentiable then it is concave if and only if

$$d^2z = f_{xx}dx^2 + 2f_{xy}dxdy + f_{yy}dy^2 \leq 0.$$