[Glossary]

[Eigenvalues and Quadratic Forms]

Subject:	Business Economics
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Paper No. & Title:	631 (Six Three One) Advanced Mathematical Techniques
Unit No. & Title:	Unit - 1 Linear Algebra
Lecture No. & Title:	1 (One) Eigenvalues and Quadratic Form

Glossary:

Characteristic polynomial of a square matrix
Given a square matrix A or order n, its characteristic polynomial is the n^{th} degree polynomial

$$|A - \lambda I|.$$

Characteristic equation of a square matrix
Given a square matrix A or order n, its characteristic equation is

$$|A - \lambda I| = 0.$$

Eigenvalues and Eigenvectors (Characteristic roots and Characteristic vectors) of a square matrix
Let A be a square matrix of order n. A real number λ is called an eigenvalue of the matrix A if there exists a non-zero (column) vector $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ in R^n satisfying $Ax = \lambda x$ The vector x is called an eigenvector of the matrix A *associated to the eigenvalue λ.*

Symmetric matrix
A square matrix $A = [a_{ij}]$ is said to be symmetric if $A = A^T$ or in other words if $a_{ij} = a_{ji}$ for all i and j.

Non-singular or Invertible matrix
Let A be a square matrix. Then A is said to be non-singular or invertible if there exists a matrix B such that $AB = BA = I$. Here B is called the inverse of A and it is denoted by A^{-1}.

Orthogonal matrix
Let A be a square matrix. Then A is said to be orthogonal if $A^{-1} = A^T$

Diagonal matrix:
A square matrix $A = [a_{ij}]$ is said to be a diagonal matrix if all its non-diagonal elements are zero or in other words if

$$a_{ij} = 0 \text{ whenever } i \neq j.$$

Similar Matrices:
Let A and B be square matrices of the same order. If there exists an invertible matrix P such that $P^{-1}AP = B$ then we say that A and B are similar matrices.

Trace of a matrix

For a square matrix $A = [a_{ij}]$ *its trace is defined as the sum of its diagonal elements.*

Linearly independent vectors

Let $v_1, v_2, \dots, v_k$ *be vectors of* $\mathbb{R}^n$*. These* k *vectors are termed as linearly independent if there does not exist real scalars* $\alpha_1, \alpha_2, \dots, \alpha_k$ *(not all zero) such that*
$\alpha_1 v_1 + \alpha_2 v_2 + \dots + \alpha_k v_k = 0.$

Orthogonal and Orthonormal vectors

Consider two vectors $x = (x_1, x_2, \dots, x_n)$ *and* $y = (y_1, y, \dots, y_n)$ *of* $\mathbb{R}^n$*. These two vectors are called orthogonal if their inner product is zero or in other words if*
$x^T y = x_1 y_1 + x_2 y_2 + \dots + x_n y_n = 0.$
Additionally if $x^T x = y^T y = 1$ *then the vectors are called orthonormal.*

Rank of a matrix

Rank of a matrix is defined as the number of linearly independent row or column vectors.

Diagonalizable matrix

Let A *be a square matrix. Then it is called a diagonalizable matrix if it is similar to a diagonal matrix or in other words if we can find an invertible matrix* P *such that* $P^{-1}AP$ *is a diagonal matrix.*

Quadratic form

Consider a square matrix $A = [a_{ij}]$ *of order* n *and a column vector* $x = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$ *in* $\mathbb{R}^n$*. Then the quadratic form associated with the matrix* $A = [a_{ij}]$ *is an expression of the form*

$$q(x) = x^T A x = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} x_i x_j.$$

Hessian matrix

For a real-valued function $f(x_1, x_2, \ldots, x_n)$ if all its second order partial derivatives exist and are continuous over the domain of the function, then its Hessian matrix is a square matrix of order n given by

$$H = [f_{ij}]$$

where $f_{ij} = \frac{\partial^2 f}{\partial x_i \partial x_j}$.