[Frequently Asked Questions]

[Eigenvalues and Quadratic Forms]

Subject:	Business Economics
Course:	B.A., 6th Semester, Undergraduate
Paper No. & Title:	631 (Six Three One) Advanced Mathematical Techniques
Unit No. & Title:	Unit - 1 Linear Algebra
Lecture No. & Title:	1 (One) Eigenvalues and Quadratic Form

Frequently Asked Questions

Q.1 Does there exist a matrix without any real eigenvalues?
A.1 Yes. Consider the matrix $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$. Its characteristic equation is $\lambda^2 + 1 = 0$ which has no real roots.

Q.2 For a square matrix of order n, is it possible that all its eigenvalues are identical?
A.2 Yes. Just consider a diagonal matrix all whose diagonal elements are identical.

Q.3 For a square matrix of order n, is it possible that every non-zero vector in $\mathbb{R}^n$ is its eigenvector?
A.3 Yes. Just consider a diagonal matrix all whose diagonal elements are identical.

Q.4 Can we say that the sum of two eigenvectors associated with the same eigenvalue of a square matrix A, is also an eigenvector of A?

A.4 Yes. Suppose x and y are two eigenvectors associated with some eigenvalue λ. Then for any real scalars α and β, the vector $\alpha x + \beta y$ is also an eigenvector of A associated with the eigenvalue λ because

$$A(\alpha x + \beta y) = \alpha A x + \beta A y = \alpha \lambda x + \beta \lambda y = \lambda(\alpha x + \beta y).$$

Q.5 How can we say that the eigenvalues of A and A^T are same?
A.5 This is because
$|(A - \lambda I)| = |(A - \lambda I)^T| = |(A^T - \lambda I)|$
and so the characteristic equations of A and A^T are same.

Q.6 How do we show that the trace of the two similar matrices is equal?
A.6 Use the fact that matrix multiplication is associative and that trace

of AB=trace of BA

for any two square matrices A and B of the same order.

Q.7 How do we show that the determinant of the two similar matrices is equal?
A.7 Use the fact that matrix multiplication is associative and that $|A^{-1}| = \frac{1}{|A|}$ for any invertible matrix A.

Q.8 How do we show that similar matrices have same set of eigenvalues?
A.8 We need to show that similar matrices have same characteristic equation and this is evident from the following identity for any square matrix A and an invertible matrix P.
$$|(A-\lambda I)| = |P^{-1}(A-\lambda I)P| = |P^{-1}AP-\lambda I|$$

Q.9 Can a square matrix of order n have more than n eigenvalues?
A.9 No. Eigenvalues are actually the roots of the characteristic polynomial of the given matrix and since the degree of this polynomial is n, it cannot have more than n roots according to the Fundamental theorem of algebra.

Q.10 Can we say that the columns of an orthogonal matrix are mutually orthogonal?
A.10 Yes. In fact it is known that a square matrix is orthogonal if and only if its column vectors form an orthonormal set.