[Frequently Asked Questions]

Generalised Least Squares

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Unit No. & Title:	Unit – 1 Relaxing the Assumptions of The Classical Linear Model
Lecture No. & Title:	Lecture – 4 Generalised Least Squares

Frequently Asked Questions

Q1. What are Generalized Least Square Estimators?

A1. In the situation when the homoscedastic situation is not valid and we find that each of disturbances have different variances and co-variances, we cannot use OLS estimators. Instead we use Generalized Least Squares Estimators.

Q2. What is Heteroscadasticity?

A2. It is absence of homoscedasticity. Thus the basic assumption about dispersion matrix is violated here. We write $V(U_i) = \sigma_i^2, i = 1,2,\ldots,n$ and $Cov(U_i, U_j) \neq 0$, $i,j = 1,2,\ldots n, i \neq j$.

This situation is called heteroscadasticity.

Q3. What will happen if we use OLSE instead of using GLSE when there is heteroscadasticity?

A3. We get estimators which are unbiased and also consistent but their variances will be larger thus giving larger standard errors which leads to misleading and faulty conclusions.

Q4. Give formulae for GLSE and its variances in K variate model

A4. For K variate model, $\underline{Y}=X\underline{\beta}+\underline{U}$ where $\underline{Y}$: nx1, X: nxk, $\underline{\beta}$: kx1, $\underline{U}$: nx1, GLSE for $\underline{\beta}$ are given by $\underline{\hat{\beta}_g} = (X'VX)^{-1}\ X'V^{-1}\underline{Y}$

and its dispersion matrix is given by

$V\left(\underline{\hat{\beta}_g}\right) = (X'V^{-1}X)^{-1}\sigma^2$

[If we write $\sigma^2 V = \Omega$, the above relations are

$\underline{\hat{\beta}_g} = (X'\Omega^{-1}X)^{-1}\left(X'\Omega^{-1}\underline{Y}\right)$

And $V\left(\underline{\hat{\beta}_g}\right) = (X'\Omega^{-1}X)^{-1}$

Q5. What are Aitkin's estimators?

A5. GLSE are also called Aitkin's estimators.

Q6. What are Harder and Milder forms of heteroscadasticity?

A6. If $V(U_i) = \sigma_i^2 = \sigma^2 X_i^2$ (and similar higher powers for X_i) it is called Harder heteroscadasticity. Here variance of disturbance term is proportional to square of explanatory variable.

If $V(U_i) = \sigma_i^2 = \sigma^2 X_i$ (and similar results like $\sigma^2\sqrt{X_i}$ etc) it is called milder heteroscadasticity. Here variance of disturbance term is proportional to the explanatory variable.

Q7. What are Weighted Least Square Estimators?

A7. Under the presence of heteroscadasticity when $V(U_i) = \sigma_i^2$ are known, we use weighted least squares methods to estimate the regression coefficients. They are called WLSE (Weighted Least Squares Estimators). Here the advantage is that larger weight is given to smaller value of disturbance variance and smaller weight is given when the variances are larger. This is due to the fact that for finding weighted average, weights are taken inversely proportional to the variances of disturbance terms.

Q8. While doing data analysis how unknowingly we have to face the problem of heteroscadasticity?

A8. This occurs due to some reasons like

(a) Grouping of observations

(b) Grouping of equations

(c) By imposing linear stochastic constraints upon the parameters.

All such cases should be dealt with by means of using GLSE instead of OLSE.

Q9. How can we use Rank correlation test to detect heteroscadasticity?

A9. We run the model as usual and find $|e_i|$. Then obtain rank correlation coefficient between $|e_i|$ and X_i. Let it be r_s then apply t test for $H_0 : \rho_s = 0$ against $H_1 : \rho_s \neq 0$

(ρ_s = population rank correlation coefficient)

Apply t test where $t = \frac{r_s \sqrt{n-2}}{\sqrt{1-r_s^2}}$ with (n-2) degrees of freedom

If t is significant, we conclude that there is heteroscadasticity, not otherwise.

Q10. Why logarithmic transformation method is popular in dealing with heteroscadastic situation?

A10. By taking log values, the actual figures will be reduced, thus decreasing variation and hence easing for heteroscadastic situation.

Also by considering logarithmic model, the coefficient attached to slope will give elasticity measure which is good for examining the sensitivity of the slope coefficient.