ASSIGNMENT

1. Prove that limit of a sequence if it exists is unique.

2. Let $\{x_n\}$ and $\{y_n\}$ be convergent sequences. Then show that

a) $\lim\limits_{n\to\infty}(x_n \pm y_n) = \lim\limits_{n\to\infty} x_n \pm \lim\limits_{n\to\infty} y_n$.

b) $\lim\limits_{n\to\infty}(x_n y_n) = (\lim\limits_{n\to\infty} x_n)(\lim\limits_{n\to\infty} y_n)$.

c) $\lim\limits_{n\to\infty}(x_n/y_n) = \lim\limits_{n\to\infty} x_n \,/\, \lim\limits_{n\to\infty} y_n$, provided $\lim\limits_{n\to\infty} y_n \neq 0$.

d) $\lim\limits_{n\to\infty}(\alpha\, x_n) = \alpha \lim\limits_{n\to\infty} x_n \; for\ every\ real\ \alpha$.

3. Prove that every convergent sequence is bounded.

4. If $p(x)$ and $q(x)$ are polynomials with leading coefficients as $l > 0$ and m respectively then show that

a) $\lim\limits_{n\to\infty} \frac{p(n)}{q(n)} = \frac{l}{m}$ if $p(x)$ and $q(x)$ are of same degree.

b) $\lim\limits_{n\to\infty} \frac{p(n)}{q(n)} = 0$ if degree of $q(x)$ exceeds that of $p(x)$.

c) $\lim\limits_{n\to\infty} \frac{p(n)}{q(n)} = \infty$ if degree of $p(x)$ exceeds that of $q(x)$.

5. If the series $\sum_{n=1}^{\infty} x_n$ and the series $\sum_{n=1}^{\infty} y_n$ are convergent then prove that the series $\sum_{n=1}^{\infty}(x_n \pm y_n)$ is convergent and

$$\sum_{n=1}^{\infty}(x_n \pm y_n) = \sum_{n=1}^{\infty} x_n \pm \sum_{n=1}^{\infty} y_n \,.$$

6. If the series $\sum_{n=1}^{\infty} x_n$ converges and $\alpha \in \mathbb{C}$, then prove that the series $\sum_{n=1}^{\infty}(\alpha x_n)$ converges and

$$\sum_{n=1}^{\infty} \alpha x_n = \alpha \sum_{n=1}^{\infty} x_n .$$