[Academic Script]

[Derivatives (Part - 2)]

Subject:	Business Economics
Course:	B. A. (Hons.), 5th Semester, Undergraduate
Paper No. & Title:	Paper – 511 Macroeconomics - II
Unit No. & Title:	Unit – 5 Derivatives
Lecture No. & Title:	Lecture – 2 Derivatives (Part - 2)

DERIVATIVES (PART-2)

INTRODUCTION

An option contract is a contract, which is actively traded on an exchange and derives its value from an underlying asset like stock and stock indices. There are two parties in an option contract the option seller also known as the writer of the contract and option buyer.

The option buyer has the right, but no obligation, to buy (sell) an asset at the exercise price from (to) the option seller (buyer) within a specified time period.

OPTIONS

MEANING

An option contract is a contract, that, in exchange for the option price, gives the option buyer the right , but no obligation, to buy (sell) an asset at the exercise price from (to) the option seller (buyer) within a specified time period, or depending on the type of option, a precise date (i.e. expiration date).

A **Call** option gives the option holder the right to purchase the underlying asset by a certain specified date for a specified date for a specified (in advance) price.

A **Put** option gives the option holder the right to sell the underlying asset on a selected date for pre-selected price.

The price in the contract is known as the ***exercise price*** or ***strike price (K)***; the date in the contract is known as the ***expiration date*** or ***maturity (t)*** and current price of the underlying asset is the ***spot price (S)***.

American options can be exercised at any time up to the expiration date. **European options** can be exercised only on the expiration date itself. ***Most of the options that are traded on exchanges are American***. In the exchange-traded equity option market, one contract is usually an agreement to buy or sell 100 shares. European options are generally easier to analyze than American options, and some of the properties of an American option are frequently deduced from those of its European counterpart.

OPTION POSITION:

There are two sides to every option contract. On one side is the investor who has taken the long position (i.e., has bought the option). On the other side is the investor who has taken a short position (i.e., has sold or written the option). The writer of an option receives cash up front, but has potential liabilities later. The writer's profit or loss is the reverse of that for the purchaser of the option.

There are four types of option positions:

- A long position in a call option
- A long position in a put option
- A short position in a call option
- A short position in a put option.

Options are referred to as in the money, at the money, or out of the money.

IN THE MONEY – AT THE MONEY – OUT OF MONEY OPTIONS

Call Option: If S is the stock price and K is the strike price, a call option is ***in the money*** when $S > K$, ***at the money*** when $S = K$, and out of the money when $S < K$.

Put option: A put option is ***in the money*** when $S < K$, at the money when $S = K$, and out of the money when $S > K$. Clearly, an option will be exercised only when it is in the money. In the absence of transactions costs, an in-the-money option will always be exercised on the expiration date if it has not been exercised previously.

ILLUSTRATION

	Call Option	**Put Option**
In the money	If K = 100, and S = 110 i.e. S > K	If K = 100 and S = 90 i.e. S < K
At the money	If K =100 and S = 100 i.e. S = K	If K =100 and S = 100 i.e. S = K
Out of the money	If K = 100 and S = 90 i.e. S < K	If K = 100, and S = 110 i.e. S > K

CALL OPTION PAYOFF

The payoff on a call option to the option buyer is calculated as follows

$$C_T = \max(0, S_T – X)$$

Where

C_T – Payoff on call option

S_T - Stock price at maturity

X – Strike price of option

The payoff to the option seller is $-C_T$ [i.e. , $- \max(0,S_T – X)$]. We should note that $\max(0,S_T – X)$, where time, *t*, is between 0 and T, is also the payoff if the owner decides to exercise the call option early.

The price paid for the call option, C_0 , is referred to as the **call premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{Profit} = C_T - C_0$$

Where:

C_T = payoff on call option ; C_0= call premium

Conversely, the profit to the option seller is: **Profit = $C_0 - C_T$**

PUT OPTION PAYOFF

The payoff on a put option is calculated as follows

$$P_t = \max(0, X-S_t)$$

Where

P_t = payoff on put option

S_t = Stock price at maturity

X = Strike price of option

The payoff to the option seller is $–P_t$ [i.e. $\max(0, X-S_t)$]. We should note that $\max(0, X-S_t)$, where $0<t<T$, is also the payoff if the owner decides to exercise the put option early.

The price paid for the put option, P_0, is referred to as the **put premium**. Thus, the profit to the option buyer is calculated as follows:

$$\text{Profit} = P_t-P_0$$

Where:

Pt = payoff on put option; P_0 = put premium

The profit to the option seller is: **Profit = $P_0 - P_t$**

TRADING STRATEGIES INVOLVING OPTIONS

Traders and investors use option-based trading strategies to create an extraordinary spectrum of payoff profiles. This enables investors to take positions based on almost any possible expectation of the underlying stock over the life of the options.

Below are the options trading strategies:

- **Covered Call:**

When a long position on the underlying stock is combined with a short position on a out of the money call option a covered call strategy can be constructed. By writing an out of the money call option, the combined position caps the upside potential at the strike price. In return for giving up any potential gain beyond the strike price, the writer receives the option premium. This strategy is used to generate cash on a stock that is not expected to increase above its exercise price over the life of the option.

- **Protective Put:**

When at the money put long position is combined with the underlying stock, we have created a protective put strategy. A protective put is constructed by holding a long position in the underlying security and buying a protective put option. This is used to limit the downside risk at the cost of the put option premium, P_0.

- **Spread Strategies:**

These strategies combine option positions to create a desired payoff profile. The differences between these options are either strike price, and or time to expiration. They are as below

1. **Bull and Bear Spreads:** In a ***bull call spread*** the buyer of the spread purchases a call option with a low exercise price, X_L and subsidizes the purchase price of the call by selling a call with a higher exercise price X_H. The buyer of the call spread expects the stock price to rise and the purchased call to finish in-the-money. However, the buyer does not believe that the price of the stock will rise above the exercise price for out of the money written call. In a ***bear call spread*** is the sale of a bull spread. That is the bear spread trader will purchase the call with the higher exercise price and sell the call with the lower exercise price. This strategy is designed to profit from falling stock prices. As stock prices fall, the investor keeps the premium from the written call, net of the long calls cost. The purpose of the long call is to protect from sharp increase in the stock prices.

2. **Butterfly Spreads:** A butterfly spread involves the purchase or sale of three different options. The investor buys one call with low exercise price, buys another call with a high exercise price, and sells two calls with an exercise price in between. The buyer is essentially betting that the stock price will remain near the strike price of the written calls. However, the loss that the butterfly spread buyer sustains of the stock price strays from this level is limited.

3. **Calendar Spreads:** A calendar spread is created by transacting in two options that have same strike price but different expirations. The investor profits only if the stock remains in a narrow range, but losses are limited.

4. **Diagonal Spreads:** A diagonal spread is similar to a calendar spread except that instead of using options with same strike price and different expiration, the options in diagonal spread can have different strike prices in addition to different expiration.

5. **Box Spreads:** A Box spread is a combination of a bull call spread and a bear put spread on the same asset. The payoff to the box spread is always the same, so if the options are priced correctly, the pay-off must be the risk free rate.

4. **Combination Strategies:** A combination is an option trading strategy that involves taking a position in both calls and puts on the same stock.

1. **Straddle:** One popular combination is a straddle, which involves buying a European call and put with the same strike price and expiration date. If the stock price is close to this strike price at expiration of the options, the straddle leads to a loss. However, if there is a sufficiently large move in either direction, a significant profit will result. A straddle is appropriate when an investor is expecting a large move in a stock price but does not know in which direction the move will be.

2. **Strips and Straps:** A strip consists of a long position in one European call and two European puts with the same strike price and expiration date. A ***strap*** consists of a long position in two European calls and one European put with the same strike price and expiration date. In a ***strip*** the investor is betting that there will be a big stock price move and considers a decrease in the stock price to be more likely than an increase. In a strap the investor is also betting that there will be a big stock price move. However, in this case, an increase in the stock price is considered to be more likely than a decrease.

3. **Strangles:** In a strangle, sometimes called a *bottom vertical combination*, an investor buys a European put and a European call with the same expiration date and different strike. A strangle is a similar strategy to a straddle. The investor is betting that there will be a large price move, but is uncertain whether it will be an increase or a decrease. The profit pattern obtained with a strangle depends on how close together the strike prices are. The farther they are apart, the less the downside risk and the farther the stock price has to move for a profit to be realized. The sale of a strangle is sometimes referred to as a *top vertical combination*. It can be appropriate for an investor who feels that large stock price moves are unlikely. However, as with sale of a straddle, it is a risky strategy involving unlimited potential loss to the investor.

PUT - CALL PARITY

We will now derive an important relationship between the prices of European put and call options that have the same strike price and time to maturity. Consider the following two portfolios

Portfolio A:	One European call option plus a zero-coupon bond that provides a payoff of K at time T	This is equal to a covered call strategy
Porfolio C:	One European put option plus one share of the stock	This is equal to a protective put strategy

We assume that the stock pays no dividends. The call and put options have the same strike price K and the same time to maturity T. The zero-coupon bond in portfolio A will be worth K at time T.

If the stock price S_T at time T proves to be above K, then the call option in portfolio A will be exercised. This means that portfolio A is worth $(S_T - K) + K = S_T$, at time T.

If S_T proves to be less than K, then the call option in portfolio A will expire worthless and the portfolio will be worth K at time T.

In portfolio C, the share will be worth S_T at time T. If ST proves to be below K, then the put option in portfolio C will be exercised.

This means that portfolio C is worth $(K – S_T) + S_T = K$ at time T.

If S_T proves to be greater than K, then the put option in portfolio C will expire worthless and the portfolio will be worth S_T at time T.

If $S_T > K$, both portfolios are worth S_T at time T; if $S_T < K$, both portfolios are worth K at time T.

Because they are European, the options cannot be exercised prior to time T. Since the portfolios have identical values at time T, they must have identical values today. If this were not the case, an arbitrageur could buy the less expensive portfolio and sell the more expensive one. Because the portfolios are guaranteed to cancel each other out at time T, this trading strategy would lock in an arbitrage profit equal to the difference in the values of the two portfolios. The components of portfolio A are worth c and Ke^{-rt} today, and the components of portfolio C are worth p and S0 today.

Hence,

$$\mathbf{c + Ke^{-rt} = p + S_0}$$

This relationship is known as put–call parity. It shows that the value of a European call with a certain exercise price and exercise date can be deduced from the value of a European put with the same exercise price and exercise date, and vice versa.

FACTORS AFFECTION OPTION PREMIUM

The following six factors impact the value of an option:

- S_0 – Current Stock Price
- X – Strike price of the option
- T – Time to expiration of the option
- r – Short term risk free interest rate over *T*
- D – Present value of the dividend of the underlying stock
- σ - Expected volatility of stock prices over *T*

When evaluating a change in any one of the factors, hold the other factors constant:

Current Stock Price: For Call Option, as S increases (decreases), the value of call increases (decreases). For put option, as S increases (decreases), the value of put decreases (increases). This simply means that, as an option becomes closer to or more in the money, its value increases.

Strike Price of the option: For call option, as X increases (decreases), the value of the call decreases (increases). For put options, as X increases (decreases) value of the put increases (decreases). Thus, the option value will increase as it becomes closer to or more in the money.

Time to expiration of the option: For American style options, increasing time to expiration will increase the option value, as with more time the likely hood of being the money increases.

Short term risk free rate over the life of the option: As the risk free rate increases, the value of the call (put) will increase (decrease).

Dividends: The option owner does not have access to the cash flows of the underlying stock, and the stock price decreases when a dividend is paid. Thus, as dividend increases, the value of the call (put) will decrease (increase)

Volatility of the stock price over the life of an option: Volatility is the friend of the option. As volatility increases, option value increases. This is due the asymmetric payoff of the options, since long option positions have a maximum loss equal to the premium paid, increased volatility only increases the chances that the option will expire in-the-money. Thus it is considered to be the most important factor in option valuation.

Summary of effects of increasing a factor on the price of an option

Factor	European Call	European Put	American Call	American Put
S	+	-	+	-
X	-	+	-	+
T	N.A.	N.A.	+	+
Σ	+	+	+	+
R	+	-	+	-
D	-	+	-	+

OPTION PRICING MODEL

A useful and very popular technique for pricing an option involves constructing a binomial tree. This is a diagram representing different possible paths that might be followed by the stock price over the life of an option. The underlying assumption is that the stock price follows a random walk. In each time step, it has a certain probability of moving up by a certain percentage amount and a certain probability of moving down by a certain percentage amount.

Option pricing using a Binomial Tree

The binomial model can be viewed as a discrete equivalent to the geometric Brownian motion. As before, we subdivide the horizon T into n intervals $\Delta t = T/n$. At each "node," the price is assumed to go either up with probability p, or down with probability $1 - p$.

ONE STEP BINOMIAL MODEL

The one step binomial model is best described within a two state world where the price of a stock will either go up once or down once, and the change will occur one step ahead at the end of the holding period.

THE REPLICATING PORTFOLIO

The replicating portfolio is the key to understanding how to value options. In general, the replicating portfolio is a concept that holds that the outlay for a bankruptcy-free stock position should be the same as the outlay for a call position with the same payoff. To see how this works, let's define some terms

P = the stocks current price

X = the call option's exercise price

T = the time to option expiration

I = the risk free interest rate

Su = the stock value in up state

S_d = the stock value in down state

C = the value of the call option today

ILLUSTRATION

Calculate the value of the call option where P = $100, X = $125, t = 1 year, I = 8%, Su = $200 and Sd = $50.

One period Binomial Tree

P = $100 → Su= $200 ; Sd = $50

c = → Cu= max ($0, $200-$125) = $75 ; Cd= max ($0, $50- $125) = $0

Value of the option: Thus the price of the option must be $75.

Calculation of Upward downward movement

$$U = \text{size of the up-move factor} = e^{\sigma\sqrt{t}}$$

$$D = \text{size of the down-move factor} = e^{-\sigma\sqrt{t}} = \frac{1}{e^{\sigma\sqrt{t}}} = \frac{1}{U}$$

where:
σ = annual volatility of the underlying asset's returns
t = the length of the step in the binomial model

<u>Calculation of the risk neutral probability of upward or downward movements are then calculated as follows</u>

$$\pi_u = \text{Probability of an up move} = \frac{e^{rt} - D}{U - D}$$

$$\pi_d = \text{Probability of a down move} = 1 - \pi_u$$

Where:
r = Continuously compounded annual risk-free rate

<u>TWO STEP BINOMIAL MODEL</u>
In the two period and multi period models, the tree is expanded provided for a greater number of potential outcomes. The stock price tree for the two period model is shown as below

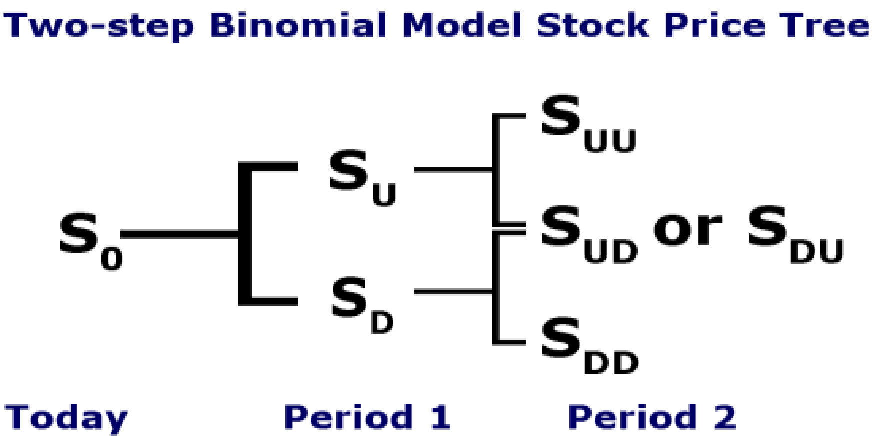


ILLUSTRATION
Option valuation using a two step binomial model where P = $20, R = 4%, U = 1.15, D = 0.87, $\pi_u = 0.61$, $\pi_d = 0.39$ and annual standard deviation is 14%.

STEP 1: CALCULATION OF UP MOVE AND DOWN MOVE FACTORS

$$U = e^{0.14 \times \sqrt{1}} = 1.15$$
$$D = 1 / 1.15 = 0.87$$

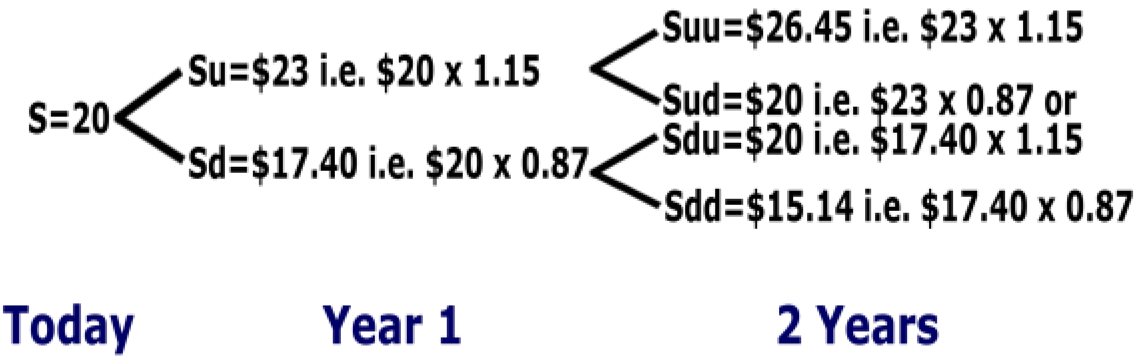
STEP 2: THE RISK NEUTRAL PROBABILITY OF AN UP MOVE AND DOWN MOVE IS:

$$\pi_u = \frac{e^{0.04 \times 1} - D}{U - D} = \frac{1.0408 - 0.87}{1.15 - 0.87} = 0.61$$

$$\pi_d = 1 - 0.61 = 0.39$$

The Binomial tree for the Stock is as below

The Binomial Tree for the stock is as



From this, the values of call option in each of the possible outcomes can be determined.

Notice that **<u>the only time that the option is in the money</u>** is when two upward price movements lead to an ending price of $26.45 and a call value of $6.45. The expected value of the option at the end of year 2 is the value of the option in each state multiplied by the probability of that state occurring.

Expected call value in 2 years = (0.61 x 0.61 x $6.45) + (0.61 x 0.39 x $0)
+ (0.39 x 0.61 x $0) + (0.39 x 0.39 x $0)
= (0.3721 x $6.45)
= **$ 2.40**

The value of the option today is the expected value in two years discounted at the risk free rate of 4%

Call option Value = $2.40 / $e^{(0.04) \times (2)}$ = $2.21

CONCLUSION

The above can be summarized as options are mainly derivatives contracts in exchange for a price and it derives its value from an underlying stock. There are two types of option call option and put option each having two positions a long position and a short position. American options and European options are two main types of option. However, we commonly deal with European options.

The premium of the option contracts is contingent upon whether the option is in the money, At the money and Out of the money. In order to hedge the risk many traders, investors and arbitrators use various option trading strategies like protective put, covered call, spread and combination strategies.