Linear Programming Problem Formulation & Inter Predation (part-2)

Subject	**:**	**Business Economics**
Course	**:**	**B.A., 5th Semester, Undergraduate**
Paper No.	**:**	**502**
& Title	**:**	**Computational Techniques for Business Economics**
Topic No.	**:**	**4**
& Title	**:**	**Linear Programming And mathematical Problem solving**
Title	**:**	**Linear Programming Problem Formulation & Inter Predation (part-2)**

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Introduction

In previous sessions having learnt the basics of LPP we went ahead to understand the formulation part of LPP in which taking an example of lumber mill problem we formulated the LPP mathematically and also to simplify the analysis part we started with the graphical solution of the problem. Having understood how to plot the points of constraints on the graph today we will go further to discuss the feasible region and solution. The graphical solution method can only be applied to LP problems with two variables. For problems that are larger than this, we will rely on the computer to provide solutions. Now the output that computer gives is much more than only optimal solution like reduced cost, Slack or surplus and Dual Price/ shadow price. Hence we will be understanding those too.

Let me quickly revise the The basic steps in formulation so as to understand today's session better. They are:

1. Identify the decision variables;
2. Formulate the objective function; and
3. Identify and formulate the constraints.
4. A trivial step, but one should not forget, is writing out the non-negativity constraints.

The only way to learn how to formulate linear programming problems is to do it. So, considering the lumber mill problem example, and the points already plotted let us start with its solution part.

Feasible region solution

As shown in the figure on the screen all the constraints have been identified and plotted along with the non-negativity rule. The constraints form a closed polygon containing all of the feasible solutions to the problem. Any point inside this polygon satisfies all of the constraints. This polygon is called the *feasible region*. Because the feasible region for this problem actually contains some points, we can conclude that there are feasible solutions to this problem – i.e., there are points, corresponding to different values of *L*and *P*, that satisfy all of the constraints. Also, the fact that the feasible region is closed indicates that the problem is not unbounded.

The problem now is to find the point (or set of points) within the feasible region that produces the highest value of the objective function. Unlike the constraints, the objective function does not correspond to a single line. Instead, it defines a series of parallel lines, each corresponding to a different value of the objective function. For example, the value of the objective function can be arbitrarily set at 3,000. That is, let

$$Z = 3{,}000 = 3 \cdot P + 10 \cdot L$$

The same techniques used earlier to graph the constraints can be used to graph this line. This line crosses the x-axis at the point (300,0) and the y-axis at the point (0, 1,000). Note where this line is in Figure on screen. It crosses the feasible region between the points (120, 600) and (200, 333.3). (*Could you have identified these points?*) Any of the points on the line segment between these two points is feasible and will give an objective function value of 3,000.

Is 3,000 the highest objective function value that can be achieved? What happens if the objective function is set at a higher level, say 4,000. All of the points producing an objective function of 4,000 will fall on the following line:

$$Z = 4{,}000 = 3 \cdot P + 10 \cdot L$$

This line crosses the x-axis at the point (400, 0), and it crosses the y-axis at the point (0, 1,333.3). Locate the line in Figure on the screen. This line does not intersect the feasible region at any point. This means that there are no points that are feasible that give an objective function value of 4,000.

Note that the two lines identified by setting the objective function value to 3,000 and 4,000, respectively, are parallel. Each value that the objective function can take will correspond to a line on the graph. All of these lines defined by different values of the objective function will be parallel. Note also that as the objective function value increases, the line defined by theobjective function moves farther out from the origin of the graph (the point (0,0)). All of the space between the two objective function lines discussed so far can be filled with parallel lines corresponding to different values of the objective function between 3,000 and 4,000. The bestpossible value of the objective function will correspond to the line that is as far from the origin as possible that still touches at least one point in the feasible region.

Imagine sliding the objective function line out from the origin by gradually increasing the objective function value. These lines will all be parallel to the lines that have already been drawn. Eventually, the line will move beyond the feasible region. The last point in the feasible region that is touched by the objective function as it is moved away from the origin is the optimal solution. The last feasible point that the line will touch will be one of the corners, or possibly the line segment (or *face*) between two corners. If the last point that is touched is a corner, then that corner is the optimal solution. If the line touches a face last, then every point on that face, including the two corners at the ends of the face, will be equally good. In either case, one or more of the corners will be in the optimal solution.

You can see by looking at the graph that the solution to the lumber mill problem will be at the corner where the pallet capacity constraint intersects the log capacity constraint. This corner corresponds to the optimal solution. Because this point is on the pallet capacity constraint, the value of P at this corner must be 600. The value of L at this point can be identified by setting P equal to 600 in the log capacity constraint. The values of the variables at this corner are $P = 600$ and $L = 178.6$. Thus, the solution to the lumber mill problem is:

$L = 178.6$ and $P = 600$.

In other words, the production strategy that will result in the highest daily net revenue is to produce 600 pallets and 178.6 mbf of lumber per day.

The best value of the objective function is obtained by plugging the values of the variables at this corner into the objective function.

$$\mathbf{Z = 3 \cdot 600 + 10 \cdot 178.6 = 3{,}585.7}$$

The daily net revenue with this production strategy will therefore be Rs.3,585.7. This value of the objective function — 3,585.7 — gives a line that just touches the feasible region at the point (178.6, 600). This line is shown in figure on screen.

Three key points that you should have learned from the graphical solutions are:

1) The constraints should define a polygon called the feasible region;
2) The objective function defines a set of parallel lines one for each potential value of the objective function; and

3) The solution is the last corner or face of the feasible region that the objective function

touches as the value of the objective function is improved.

This third point implies two important facts.

First, the solution to a LP problem always includes at least one corner. Second, the solution is not always just a single point. If more than one corner point is optimal, then the face between those points is also optimal. The fact that the solution always includes a corner is used by the solution algorithm for solving LP problems. The algorithm searches from corner to corner, always looking for an adjacent corner that is better than the current corner. When a corner is found which has no superior adjacent corners, then that is reported as the solution. Some of the adjacent corners may be equally good, however.

Interpreting Computer Solutions of Linear Programming Problems

The graphical solution method can only be applied to LP problems with two variables. For problems that are larger than this, we will rely on the computer to provide solutions. A variety of programs have been written to solve linear programming problems. You can also solve small linear programs with a spreadsheet, such as Excel. For larger linear programming problems, you will need a more specialized program, like LINDO. Next session discusses setting up, solving, and interpreting LP problems with Excel.

As discussed earlier, the solution to an LP problem is a set of optimal values for each of the variables. However, the output that comes with the solution to a LP problem usually contains much more information than just this. In addition to the optimal values of the variables, the output will typically include *reduced cost* values, *slack or surplus* values, and *dual prices* (also known as *shadow prices*).

Reduced Cost

Associated with each variable is a *reduced cost value*. However, the reduced cost value is only non-zero when the optimal value of a variable is zero. A somewhat intuitive way to think about the reduced cost variable is to think of it as indicating how much the cost of the activity represented by the variable must be reduced before any of that activity will be done. More precisely,

> ... The *reduced cost value* indicates how much the objective function coefficient on the corresponding variable must be improved before the value of the variable will be positive in the optimal solution.

In the case of a minimization problem, “improved” means “reduced.” So, in the case of a cost-minimization problem, where the objective function coefficients represent the per-unit cost of the activities represented by the variables, the “reduced cost” coefficients indicate how much each cost coefficient would have to be reduced before the activity represented by the corresponding variable would be cost-effective. In the case of a maximization problem, “improved” means “increased.” In this case, where, for example, the objective function coefficient might represent the net profit per unit of the activity, the reduced cost value indicates how much the profitability of the activity would have to increase in order for theactivity to occur in the optimal solution. The units of the reduced cost values are the same as the units of the corresponding objective function coefficients.

If the optimal value of a variable is positive (not zero), then the reduced cost is always zero.
If the optimal value of a variable is zero and the reduced cost corresponding to the variable is also zero, then there is at least one other corner that is also in the optimal solution. The value of this variable will be positive at one of the other optimal corners.

Slack or Surplus

A *slack* or *surplus* value is reported for each of the constraints. The term “slack” applies to less than or equal constraints, and the term “surplus” applies to greater than or equal constraints. If a constraint is binding, then the corresponding slack or surplus value will equal zero. When a less-than-or-equal constraint is not binding, then there is some un- utilized, or slack, resource.

> The *slack* value is the amount of a resource, as represented by a less-than-or- equal constraint, that is not being used. When a greater-than-or-equal constraint is not binding, then the *surplus* is the extra amount over the constraint that is being produced or utilized.

The units of the slack or surplus values are the same as the units of the corresponding constraints.

Dual Prices (i.e Shadow Prices)

The *dual prices* are some of the most interesting values in the solution to a linear program. A dual price is reported for each constraint. The dual price is only positive when a constraint is binding.

> The *dual price* gives the improvement in the objective function if the constraint is relaxed by one unit.

In the case of a less-than-or-equal constraint, such as a resource constraint, the dual price gives the value of having one more unit of the resource represented by that constraint. In the case of a greater-than-or-equal constraint, such as a minimum production level constraint, the dual price gives the cost of meeting the last unit of the minimum production target.

The units of the dual prices are the units of the objective function divided by the units of the constraint. Knowing the units of the dual prices can be useful when you are trying to interpret what the dual prices mean.

<u>Summary</u>

Let us summarize today's session. Today's session continued our discussion on graphical representation of the problem of lumber mill and also interpretation of other variables in computer programs apart from the objective function and constraint results like reduced cost, slack or surplus and shadow price were discussed.Having understood how to plot the points of constraints on the graph today we learnt about the feasible region and solution. For problems that are larger than two variables the graphical solution method cannot be applied. Hence, we will rely on the computer to provide solutions. Now the output that computer gives is much more than only

optimal solution like reduced cost, Slack or surplus and Dual Price/ shadow price. Hence we understood those too.