Linear Programming problem
Formulation & Interpretation (Part 1)

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Introduction

In previous session we understand the basics of linear programming and problem solving. In today's session we will be discussing about problem formulation. This session would only focus on Problem formulation instead of problem solving as formulation is the most critical aspect of LLP. In problem formulation we will focus on—translating real-world problemsinto the mathematical equations of a linear program — and converting the solution into a graphical solution to understand it more clearly pictorially.

This section introduces you to the process of formulating linear programs. The basic steps in formulation are:

1. Identify the decision variables;
2. Formulate the objective function; and

3. Identify and formulate the constraints.
4. A trivial step, but one should not forget, is writing out the non-negativity constraints.

The only way to learn how to formulate linear programming problems is to do it. So, consider the following example problem.

But before starting I would like you to make a note that this graphical solution and the computer interpretation is a lengthy one and hence will see how to identify decision variables and objective function as well as plotting those on graph in this session whereas the graphical feasible solution and other variable interpretations would be looked into in the next session.

Example— A Lumber Mill Problem

A lumber mill can produce pallets or high quality lumber. Its lumber capacity is limited by its kiln size. It can dry 200 mbfper day. Similarly, it can produce a maximum of 600 pallets per day. In addition, it can only process 400 logs per day through its main saw. Quality lumber sells for Rs.490 per mbf, and pallets sell forRs.9 each. It takes 1.4 logs on average to make one mbf of lumber, and four pallets can be made from one log. Of course, different grades of logs are used in making each product. Grade 1 lumber logs cost Rs.200 per log, and pallet-grade logs cost only Rs.4 per log. Processing costs per mbf of quality lumber are Rs.200 per mbf,and processing costs per pallet are only Rs.5. How many pallets and how many mbfof lumber should the mill produce?

Answer: The first step in formulating this problem is defining the decision variables. In this case, this step is fairly straightforward. Ask yourself, “whatdo I need to know in order to solve the problem?” Then read the last sentence (question) in the problem description. The problem here is to determine thenumber of pallets and the number of mbf of lumber to produce each day. Thus, the decision variables will be:

P = the number of pallets to produce each day, and
L= the number ofmbf of lumber to produce each day.

The next step is to formulate the objective function. First, consider what the manager of this mill would probably want to do. A common mistake here would be to assume that the objective is to maximize the daily revenue. The problem with this objective is that it ignores the cost of production. A more appropriate objective function would be to maximize the daily net revenue, taking into account both costs and revenues.

Now, we must translate this verbal objective into a mathematical function. Remember, the objective function must be a linear function of the variables.

This means that the objective function must have the following general form:

$$\textbf{Max } Z = c_P \cdot P + c_L \cdot L$$

Now, consider the units of the variables and the parameters. The objective is to maximize daily net revenues, so the units of Z must be Rs./day. The units of P are pallets/day, and the units of L are mbf/day. Therefore, the units of c_P must be Rs./pallet, and the units of c_L must be Rs./mbf. The coefficient c_P should give the net revenue per pallet, and the coefficient c_L should give the net revenue per mbf of lumber. A pallet sells for Rs.9. The production cost per pallet is Rs.5. To calculate the raw material cost — the log cost — per pallet, note that it takes 1/4 logs to produce one pallet and that each log costs Rs.4. The net revenue per pallet is therefore:

$$c_P(\text{Rs./pallet}) = 9(\text{Rs./pallet}) - 5(\text{Rs./pallet}) - \tfrac{1}{4}(\text{logs/pallet}) \cdot 4(\text{Rs./log}) = \text{Rs.}3/\text{pallet}$$

Similarly, the net revenue per mbf of lumber is:

$$c_L(\text{Rs./mbf}) = 490(\text{Rs./mbf}) - 200(\text{Rs./mbf}) - 1.4(\text{logs/mbf})\cdot 200(\text{Rs./log}) = \text{Rs.}10/\text{mbf}$$

We have now completed the formulation of the objective function.
It is:
Max $Z\ (\text{Rs./day}) = 3\ (\text{Rs./pallet}) \cdot P\ (\text{pallets/day}) + 10\ (\text{Rs./mbf}) \cdot L\ (\text{mbf/day})$
Or, without the units:

$$\text{Max } Z = 3 \cdot P + 10 \cdot L$$

The next step is to identify and formulate the constraints. When formulating class examples, read the problem carefully to identify any statements that imply some kind of limit on what you can do.

Limits on what you can do will typically be represented by less-than-or-equal constraints.

Greater-than-or-equal constraints will typically be needed when there is a minimum level of something that is required. Of course, in a real-worldsituation, you will have to think harder because the constraints will be less obvious. Often an unrealistic solution to an initial formulation will be your best clue that some real-world constraint has been missed.

In the current example, the first constraint mentioned is the kiln size. Kiln capacity limits the total daily production of lumber to a level less than or equal to 200 mbf/day. This constraint can be written simply as:

L(mbf/day)≤200 (mbf/day)(Kiln capacity constraint)

Note that the coefficient on the variable *L*in this constraint is 1. The coefficient on *P* is 0.

The next constraint mentioned in the problem description is the maximum capacity for pallet production. Since a maximum of 600 pallets can be produced in a day, this constraint can be written as:

P(pallets/day)≤600 (pallets/day)(Pallet capacity constraint)

In this constraint, the coefficient of *P* is 1, and the coefficient of *L*is 0.

The third, and final, constraint mentioned in the problem description is the maximum number of logs that can be processed by the main saw. This constraint is more difficult to write than the previous two, where only one variable was involved and the coefficients were either 0 or 1. As with the objective function, formulating this constraint is much easier if you keep in mind the units of the right-hand-side of the constraint and the units of the variables. The units of the right-hand-side of this constraint are logs/day. Since the units of the variable *P* are pallets/day, the units of the coefficient on the pallets variable must be logs/pallet. Similarly, the units of the coefficient on the lumber variable must be logs/mbf. Knowing the units of the parameter can be a tremendous help when you are trying to figure out what a parameter value should be. In this case, we can determine from the problem description that it takes 1/4 of a log to make a pallet and that it takes 1.4 logs to make a thousand board feet of lumber. The *log capacity constraint* can be written:

$$\frac{1}{4}(\text{logs/pallet}) \cdot P(\text{pallets/day}) + 1.4(\text{logs/mbf}) \cdot L(\text{mbf/day}) \leq 400\ (\text{logs/day})$$

The only constraints remaining are the non-negativity constraints:

$$P(\text{pallets/day}) \geq 0 \quad \text{and} \quad L(\text{mbf/day}) \geq 0$$

Now, the entire linear programming problem can be written as follows:

Max *Z (Rs./day) = 3 (Rs./pallet) · P (pallets/day) + 10 (Rs./mbf) · L (mbf/day) (Objective function)*

L(mbf/day)≤200 mbf/day (Kiln capacity constraint)
P(pallets/day)≤600 (pallets/day) (Pallet capacityconstraint)

¼(logs/pallet)·P(pallets/day)+1.4(logs/mbf) ·L(mbf/day)≤400 (logs/day) (Log capacity constraint)

***P(pallets/day) ≥0* and *L(mbf/day) ≥0(Non-negativity constraints*)**

Or, without the units:

$$\text{Max } Z = 3 \cdot P + 10 \cdot L$$

Subject to:

$$L \leq 200$$
$$P \leq 600$$
$$\tfrac{1}{4} \cdot P + 1.4 \cdot L \leq 400$$
$$P \geq 0 \text{ and } L \geq 0$$

Graphical Solution of Two-Variable Linear Programming Problems

You have now seen how two word-problems can be translated into mathematical problems in the form of linear programs. Once a problem is formulated, it can be entered into a computer program to be solved. The *solution* is a set of values for each variable that:

1. Are consistent with the constraints (i.e., *feasible*), and
2. Result in the best possible value of the objective function (i.e., *optimal*).

Not all LP problems have a solution, however. There are two other possibilities:

1. There may be no feasible solutions (i.e., there are no solutions that are consistent with all the constraints), or
2. Theproblem may be *unbounded* (i.e., the optimal solution is infinitely large).

If the first of these problems occurs, one or more of the constraints will have to be relaxed. If the second problem occurs, then the problem probably has not been well formulated since few, if any, real world problems are truly unbounded.

As mentioned earlier, we will not be too concerned with how the computer solves linear programming problems. However, it is useful to solve a couple of simple problems graphically. This method only works with problems that have two variables, so obviously it has limited applicability. However, seeing the graphical representation and solution of a LPproblem will help you understand more intuitively what a LP is and how it is solved. Let’s start by solving the Lumber Mill Problem.

Example— Graphical Solution of the Lumber Mill Problem.

Graphically solve the Lumber Mill Problem that was formulated earlier.

Answer: To graphically solve a two-variable linear program, we use a graph whose axes represent the values of the two variables. Figure on the screen shows a graph where the y-axis represents the number of pallets produced per day (*P*) and the x-axis represents the number of thousand board feet (mbf) of lumber produced each day (*L*). Note that each point on this graph represents a combination of specific values of *L*and *P*. For example, the point **A** represents a combination of 150 mbf of lumber and 200 pallets.

Each point can be identified by its coordinates as follows: (*L, P*). The point **A** has the coordinates (150, 200). Each point on the graph is a potential solution to the LP problem.

The first step is graphing the constraints. The first constraint says that lumber production must be less than 200 mbf/day. In Figure on the screen, this constraint is represented by the vertical line crossing the x-axis at 200 mbf. Since the constraint is a less-than-or-equal constraint, any point on the graph to the left side of this line satisfies this constraint.

The second constraint specifies that a maximum of 600 pallets can be produced per day. This constraint is represented in the graph by a horizontal line crossing the y-axis at 600. Again, since this is a less-than-or-equal constraint, any point on the graph that is below this line satisfies this constraint.

The third constraint, the log capacity constraint, is more difficult to graph because it involves both variables. This constraint is:

$$\frac{1}{4}\cdot P + 1.4\cdot L \leq 400$$

The easiest way to graph this function is to set one variable equal to zero and solve for the value of other variable. This procedure gives you the points where the line representing the constraint crosses the x and y axes. If *P* is set equal to zero in the equation, then *L*must equal 285.7 (400 ÷ 1.4). This tells us that the constraint crosses the x-axis at the point (285.7, 0). Similarly, setting L equal to zero results in *P* = 1,600 (400 ÷ ¼). Thus, the constraint crosses the y-axis at the point (0, 1,600). With two points of the line plotted, we can draw the rest of the line by connecting the points with a ruler. Any point that is on this line or below and to the left of this line satisfies this constraint.

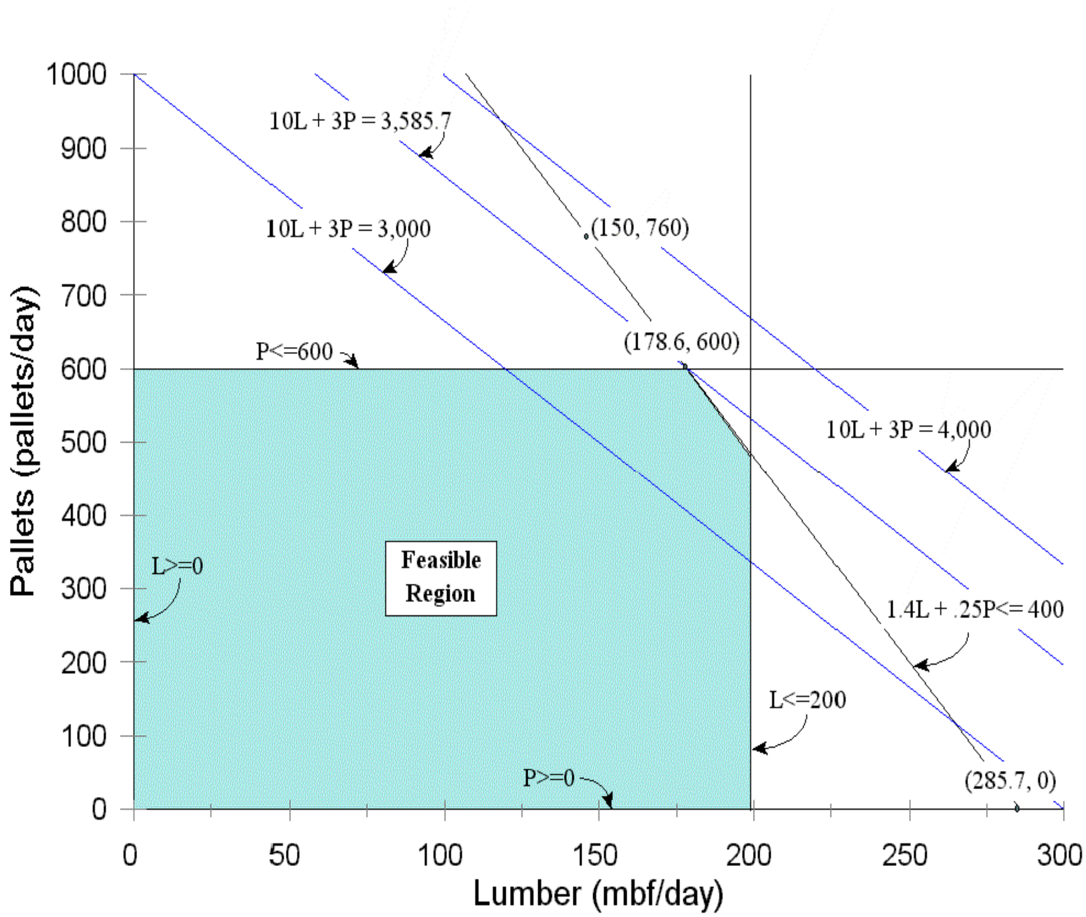

Figure 11.1. Graphical Solution of the Lumber Mill Linear Programming Problem.

Sometimes one of these points is inconvenient to plot. The second point that was just identified is an example of this. The point (0, 1,600) is too far from the main part of the graph. In these cases, it may be better to plot a different point.Insteadoffixing*L*at zero, *L*could also be fixed at 150. Plugging this value into the equation and solving for P gives P = 760 ([400 - 150·1.4] ÷¼). In this case, the point (150, 760) could be used for plotting the line instead of (0, 1,600).

In addition to these three constraints, the non-negativity constraints shouldalso be recognized on the graph. These constraints are already plotted, as they correspond to the axes themselves. Since the non-negativity constraints require the values of the variables to be positive, any values of the variables that are above the x-axis and to the right of the y-axis satisfy these constraints.

All of the constraints have now been identified on the graph. Having understood how the basic three constraints and non-negativity to be plotted on graph we will look into how to achieve feasible solution and feasible region in case of graph in the next session.

SUMMARY

Linear programming and problem solving is a very useful tool for converting real life problems into a mathematical equation helping us to solve so many daily life problems. This session taught us how to formulate a linear programming problem which is the heart of linear programing problem. We learnt the LPP formulation by identifying its objective function and various constraints through an example of lumber mill. Having formulated iot mathematically, we also saw the graphical representation of the problem wherein we identified the three constraints given in the problem and also the fundamental constraint of linear programming problem i.e. non negativity.