Introduction to Linear programming and Mathematical Problem solving

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Introduction

Hello friends, today will be a learning about linear programming and problem solving. So let us understand what basically linear programming is. LP is a mathematical technique to find optimal solutions for all those problem which can be expressed as linear equations and inequalities. If a practical life problem can be accurately represented in a mathematically linear program this is will find the best solution to it.

Definition

Linear programming can be defined as: "A mathematical method to allocate resources to competing activities in an optimal manner when the problem can be expressed using a linear objective function and linear inequality constraints"

Now let me try to simplify this definition.
A linear program consists of a set of variables, a linear objective function indicating the contribution of each variable to the desired outcome, and a set of linear constraints describing the limits on the values of the variables.

The “answer” to a linear program is a set of values for the problem variables that results in the best — largest or smallest — value of the objective function and yet is consistent with all the constraints.

Formulation is the process oftranslating a real-world problem into a linear program. Once a problem has been formulated as a linear program, a computer program can be used to solve the problem. In this regard,solving a linear program is relatively easy. The hardest and the most critical part about applying linear programming is formulating the problem and interpreting the solution.

Linear Equations

All of the equations and inequalities in a linear program must, by definition, be linear. A linear function has the following form:

$$a_0 + a_1 x_1 + a_2 x_2 + a_3 x_3 + . . . + a_n x_n = 0$$

In general, the *a*'s are called the *coefficients* of the equation; they are also sometimes called *parameters*. The important thing to know about the coefficients is that they are fixed values, based on the nature of the problem being solved.

The *x*'s are called the *variables* of the equation; they are allowed to take on a range of values within the limits defined by the Constraints.

Note that it is not necessary to always use x's to represent variables; any label could be used, and more descriptive labels are often more useful.

The linear equation above, for example, can be written as follows:

$$a_0 + \sum_{i=1}^{n} a_i x_i = 0$$

Note that the letter *i* is an *index*, or counter, starting from 1 up to n in this case. Just as a variable does not have to be specified with a letter *x*, the index does not have to be a letter *i*.

The Decision Variables

The *variables* in a linear program are a set of quantities that need to be determined in order to solve the problem; i.e., the problem is solved when the best values of the variables have been identified. The variables are sometimes called *decision variables* because the problem is to decide what value each variable should take.

Typically, the variables represent the amount of a resource to use or the level of some activity. For example, a variable might represent the number of acres to cut from a particular part of the forest during a given period. Frequently, defining the variables of the problem is one of the most crucial steps in formulating a problem as a linear program.

As mentioned earlier, a variety of symbols, with subscripts and superscripts as needed, can be used to represent the variables of an LP. As a general rule, it is better to use variable names that help you remember what the variable represents in the real world. For this general introduction, the variables will be represented — very abstractly — as X_1, X_2, . . ., X_n. (Note that there are *n* variables in this list.)

The Objective Function

The objective of a linear programming problem will be to maximize or to minimize some numerical value. This value may be the cost of a project; it could also be the amount of product produced, the amount of profit that could be earned, or the amount of a particular product to be produced from a mix of products.Thus objective function is nothing but the reason or objective behind our problem solving.

The *objective function* indicates how each variable contributes to the value to be optimized in solving the problem. The objective function takes the following general form:

$$\text{Maximize } \textit{or} \text{ minimize} \quad Z = \sum_{i=1}^{n} c_i X_i$$

The coefficients of the objective function indicate the contribution to the value of the objective function of one unit of the corresponding variable.

For example, if the objectivefunction is to maximize the present value of a project, and X_i is the *ith*possible activity in theproject, then c_i(the objective function coefficient corresponding to X_i) gives the net present value generated by one unit of activity *i*.

As another example, if the problem is to minimizethe cost of achieving some goal, $X_i$might be the amount of resource *i* used in achieving the goal.In this case, $c_i$would be the cost of using one unit of resource *i*.

Note that the way the general objective function above has been written implies that there is a coefficient in the objective function corresponding to each variable. Of course, some variables may not contribute to the objective function. In this case, you can either think ofthe variable as having a coefficient of zero, or you can think of the variable as not being in the objective function at all.

The Constraints

Constraints define the possible values that the variables of a linear programming problem may take. They typically represent resource constraints, or the minimum or maximum level of some activity or condition. They take the following general form:

$$\text{Subject to } \sum_{i=1}^{n} a_{ji}X_i \geq b_j, \quad j=1,2,\ldots m$$

Where X_i = the i^{th}decision variable.

a_{ji}= the coefficient on Xi in constraint j, and

b_j = the right-hand-side coefficient on constraint j.

Note that j is an *index* that runs from 1 to m, and each value of j corresponds to a constraint. Thus, the above expression represents m constraints (equations, or, more precisely, inequalities) with this form.

Resource constraints are a common type of constraint. In a resource constraint, the coefficient $a_{j,i}$ indicates the amount of resource j used for each unit of activity i, as represented by the value of the variable X_i. The right-hand side of the constraint (b_j) indicates the total amount of resource j available for the project.

Note also that while the constraint above is written as a less-than-or-equal constraint, greater- than-or-equal constraints can also be used. A greater-than-or-equal constraint can always be converted to a less-than-or-equal constraint by multiplying it by -1. Similarly, equality constraints can be written as two inequalities — a less-than-or-equal constraint and a greater- than-or-equal constraint.

The Non-negativity Constraints

For technical reasons the variables of linear programs must always take non-negative values (i.e., they must be greater than or equal to zero). In most cases, where, for example, the variables might represent the levels of a set of activities or the amounts of some resource used, this non-negativity requirement will be reasonable or even necessary. In any case, the non-negativity constraints are part of all LP formulations, and you should always include them in an LP formulation. They are written as follows:

$X_i \geq 0 \quad i = 1, 2. . . .n$

Where X_i is the i^{th} decision variable.

A General linear Programming Problem

At this point, all of the pieces of a general linear programming problem have been discussed. It may be useful to put all of the pieces together and look at a generalized picture of the linear programming problem. All LP problems have the following general form:

Maximize or minimize

$$Z=\sum_{i=1}^{n} C_i X_i$$

subject to

$$Z=\sum_{i=1}^{n} a_{j,i} X_i \leq b_j \quad j=1,2,\ldots,m$$

And $X_i \geq 0 \quad i = 1, 2, \ldots, n$

where X_i= the i^{th} decision variable

c_i= the objective function coefficient corresponding to the i^{th} variable,

$a_{j,\,i}$= the coefficient on *Xi* in constraint *j*, and

b_j= the right-hand-side coefficient on constraint *j*.

Note that we maximize or minimize as per the need of the situation. For example if the situation demands for profit analysis we need a maximization objective function whereas for a condition of calculating cost incurred to a company it is obvious that lower the cost, better it will be hence minimization objective function will be formulated.

The Fundamental Assumptions of Linear Programming

Now that you have seen how some simple problems can be formulated and solved as linear programs, it is useful to reconsider the question of when a problem can be realistically represented as a linear programming problem. A problem can be realistically represented as a linear program if the following assumptions hold:

1. Linearity
2. Divisibility
3. Certainty
4. Data Availability

Let us try to understand each of the assumptions.

1. The constraints and objective function are <u>linear</u>.

a) This requires that the value of the objective function and the response of each resource expressed by the constraints is proportionalto the level of each activity expressed in the variables.

b) Linearity also requires that the effects of the value of each variable on the values of the objective function and the constraints are additive.In otherwords, there can be no interactions between the effects of different activities; i.e.,thelevelofactivityX_1should not affect the costs or benefits associated withthelevelofactivityX_2.

2. Divisibility-- the values of decision variables can be fractions. Sometimes these values only make sense if they are integers; then we need an extension of linear programming called integer programming.

3. Certainty-- the model assumes that the responses to the values of the variables are exactly equal to the responses represented by the coefficients.

4. Data-- formulating a linear program to solve a problem assumes that data are available to specify the problem.

Now that you have a general idea – an abstract one – of the structure of a linear program, the next step is to consider the process of formulating a linear programming problem. The following session will explain you the process of formulating two example problems and interpreting it. This should help give you a more concrete idea of what a linear program is.

Summary

Ok friends, let us now summarize today's session. Today we learnt what linear programming is;A linear program (LP) is a mathematical formulation of a problem. We define a set of decision variables which fully describe the decisions we wish to make. We then use these variables to define an objective function which we wish to minimize or maximize, and a set of constraints which restrict the decision options open to us. In a linear program, the variables must be continuous and the objective function and constraints must be linear expressions. We also saw where it can be used and the basic concepts of linear programming like the linear equation, its objective function, its constraints, Non negativity of constraints, generalized equation to denote a linear programming problem and lastly saw the assumptions on which linear programming is based upon. I hope you would have got a macro perspective of linear programming well. Thank you,