

[Academic Script]

Decision Theory

Subject:

Course:

Paper No. & Title:

Unit No. & Title:

Business Economics

B.A., 4th Semester, Undergraduate

Paper – 403 Quantitative Techniques for Management

Unit - 4 Theory of Game, Decision Theory and Decision Analysis

Lecture No. & Title:

Lecture – 2 Decision Theory

Academic Script

1. Introduction

A decision is the process of selecting an alternative course of action, given that atleast two alternatives exists. The basic aim of decision theory is to provide a method of natural decisionmaking wherein data concerning the occurrence of different outcomes may be evaluated to allow the decision-maker to identify the best course of action.

2. Course of action (Acts)

A decision is made among a set of defined alternative course of action. These are also called actions, acts or strategies and are under control and known to the decision-maker.

For example, consider a situation where a shop-keeper has to decide how many units to be stocked, given that demand is of minimum 40 units and maximum of 80 units. The course of action for shop-keeper will be stocking 40, 41... 80 units.

3. States of nature (Event)

Outcomes of any course of action are dependent upon certain factors beyond the control of the decision-maker. These factors are called states of nature.

For example, in the above situation of stocking units for shopkeeper, the states of nature will be the actual demand of units. Demand of units is uncertain and beyond control.

4. Pay off

For each combination of an act and states of nature, there will exist an outcome. This outcome may be quantified in terms of monetary value. This outcome of act-event combination is called pay off.

States of nature	Course of action					
(Events)	A_1	A_2			A_n	
<i>E</i> ₁	<i>a</i> ₁₁	<i>a</i> ₁₂			a_{1n}	
<i>E</i> ₂	<i>a</i> ₂₁	a_{22}			a_{2n}	
:						
:						
E _m	a_{m1}	a_{m1}			a_{mn}	

A general form of a pay off matrix is shown below:

5. Steps in decision theory

The process of decision theory involves the following steps:

- List all possible future events, called states of nature, which can occur in the context of the decision problem
- Identify all the course of action which are available to the decision-maker
- Express the pay offs resulting from combination of course of action and states of nature
- Choose from among the given various course of action on the basis of some criteria that results in the optimal pay off

6. Types of decision-making

Decisions are based upon the data available about the occurrence of events as well as the decision situation. The types of decision making environment: certainty, uncertainty and risk.

(a)Decision making under certainty

In this case the decision-maker has a complete knowledge of outcome of every course of action with certainty. Clearly, he will select an act that will give the largest pay off for the known state of nature.

(b)Decision making under risk

In this case the decision-maker has less complete knowledge with certainty of outcome of every course of action. This means there is more than one states of nature and for which the decision-maker knows the probability with which each state of nature will occur.

(i) Expected Monetary Value (EMV)

Expected monetary value for a given course of action is the sum of the weighted average payoff, which is the sum of the payoffs for each course of action multiplied by the probabilities associated with each state of nature.

$$\mathsf{EMV}(A_j) = \sum a_{ij} * p_i$$

where a_{ij} = payoff associated with state of nature E_i and course of action A_j

 p_i = probability of occurrence of states of nature E_i

The course of action for which the EMV is maximum, is recommended.

(ii) Expected Opportunity Loss (EOL)

An alternative approach to maximizing EMV is to minimize the expected opportunity loss, also called expected value of regret. The EOL is the difference between the highest payoff for state of nature and the actual profit obtained for the particular course of action taken. EOL is the amount of payoff that is lost by not selecting the course of action that has the greatest payoff for the state of nature that occurs actually.

EOL $(A_j) = \sum l_{ij} * p_i$

where l_{ij} = opportunity loss associated with state of nature E_i and course of action A_i

 p_i = probability of occurrence of states of nature E_i

The course of action for which the EOL is minimum, is recommended.

(iii) Expected Value of Perfect Information (EVPI)

In decision-making under risk each state of nature is linked with the probability of its occurrence. However, if the decision maker can acquire perfect information about the occurrence of various states of nature, then he will be able to select a course of action that yields the desired payoff for whatever state of nature that actually occurs. EVPI represents the maximum amount of money the decision-maker can pay to get this additional information about the occurrence of various states of nature before a decision has to be made.

EVPI = Expected value of perfect information under certainty – Expected profit without perfect information

EVPI = $\sum a_{ij} * p_i$ - Maximum EMV

Where a_{ij}^* = best payoff associated with state of nature E_i

 p_i = probability of occurrence of states of nature E_i

(c)Decision making under uncertainty

In this case the decision-maker is unable to specify the probabilities with which the various states of nature will occur.

- (i) Maximax or Minimax Criterion
- (ii) Laplace Criterion
- (iii) Hurwicz Criterion
- (iv) Criterion of Regret (Savage Criterion)

7. Examples with Solution

Example 1: A person has three alternatives open to him each of which can be followed by any of the four possible events. The payoffs (in Rs.) for each action-event combination are given below:

Alternatives	Payoffs				
	А	В	С	D	
Х	8	0	-10	6	
Υ	-4	12	18	-2	
Z	14	6	0	8	

Determine which alternative should the person choose, if he adopts the

(i) Maximin criterion (ii) Maximax criterion (iii) Hurwicz criterion if degree of optimism is 0.7 (iv) Laplace criterion (v) Minimax regret criterion

Solution: For the given payoff matrix, the minimum and maximum possible payoffs for each alternative are as given below:

Alternative	Maximum	Minimum	∝*Maximum+(1-∝)*Minimum
	Payoff	Payoffs	
Х	8	-10	2.6
Υ	18	- 4	11.4
Z	14	0	9.8

(i) Since alternative Z yields maximum of minimum payoffs, under maximin criteria, alternative Z should be chosen.

(ii)Since alternative Y yields maximum of maximum payoffs, under maximax criteria, alternative Y should be chosen.

(iii)Since alternative Y yields maximum value of [
∝*Maximum+(1-∝)*Minimum] , under Hurwicz criteria, alternative Y should be chosen.

(**iv**)For Laplace criteria, we shall obtain average payoffs for each alternative:

Alternative X: (8+0-10+6)/4 = 1

Alternative Y: (-4+12+18-2)/4 = 6

Alternative Z: (14+6+0+8) / 4 = 7

Under Laplace criterion, alternative Z should be chosen as it gives maximum value of the average payoffs.

(**v**) For regret table, regret payoff = Maximum payoff of each event – payoff of corresponding event

Alterna		Maximum			
tives	А	В	С	D	regret
Х	14 - 8=6	12-0=12	18-(-10)=28	8-6=2	28
Υ	14-(-4)=18	12-12=0	18-18=0	8-(-2)=10	18
Ζ	14-14=0	12-6=6	18-0=18	8-8=0	18

Since alternative Y and Z both correspond to the minimal of the maximum possible regrets, the person should chose either of these two.

Example 2: The demand for a product is given below:

Demand: 40 45 50 55 60 65 Probability:0.10 0.20 0.30 0.25 0.100.05 The product costs Rs. 60 per unit and sells at Rs. 80 per unit. If the units are not sold within the season, they will have no market value. Determine the optimal number of units to be produced. Also calculate EVPI and interpret it.

Solution: Cost of the production = Rs. 60 and selling price = Rs. 80, so profit if it is sold = Rs. 80 - Rs. 60 = Rs. 20 and loss if unsold = Rs. 60 per unit.

Demand	Prob.	Strategy					
		40	45	50	55	60	65
40	0.10	800	500	200	-100	-400	-700
45	0.20	800	900	600	300	0	-300
50	0.30	800	900	1000	700	400	100
55	0.25	800	900	1000	1100	800	500
60	0.10	800	900	1000	1100	1200	900
65	0.05	800	900	1000	1100	1200	1300
	EMV	800	860	840	700	460	180

Since the EMV is highest with 45 units, the optimum number of units to be produced is 45.

Expected Profit for Perfect Information (EPPI) = 800*0.1+900*0.2+1000*0.3+1100*0.25+1200*0.1+1300*0.05 = 1020

Expected Value of Perfect Information (EVPI) = EPPI – Max. EMV = 1020 - 860 = 160.

8. Summary

• Course of action (Acts)

A decision is made among a set of defined alternative course of action. These are also called actions, acts or strategies.

States of nature (Event)

Outcomes of any course of action are dependent upon certain factors beyond the control of the decision-maker. These factors are called states of nature.

• Pay off

For each combination of an act and states of nature, there exists an outcome. This outcome of act-event combination is called pay off.

• Types of decision-making

Decisions are based upon the information data available about the occurrence of events as well as the decision situation. The types of decision making environment: certainty, uncertainty and risk.

(a)Decision making under certainty

In this case the decision-maker has the complete knowledge of outcome of every course of action with certainty.

(b)Decision making under risk

(i)Expected Monetary Value (EMV)

(ii) Expected Opportunity Loss (EOL)

(iii) Expected Value of Perfect Information (EVPI)

(c)Decision making under uncertainty

In this case the decision-maker is unable to specify the probabilities with which the various states of nature will occur.

- (i) Maximax or Minimax Criterion
- (ii) Laplace Criterion
- (iii) Hurwicz Criterion

Criterion of Regret (Savage Criterion)