[Academic Script]

Assignment

Subject:	Business Economics
Course:	B.A., 4th Semester, Undergraduate
Paper No. & Title:	Paper – 403 Quantitative Techniques for Management
Unit No. & Title:	Unit - 2 Transportation & Assignment
Lecture No. & Title:	Lecture – 3 Assignment

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1. Introduction

We have studied transportation problem in previous lectures. Transportation problem refers to allocating resources, machines, materials, capital etc. in a best possible way so that the costs are minimized or profits are maximized.

The assignment problem is the special case of the transportation problem in which the objective is to assign a number of origins to the equal number of destinations at a minimum cost, minimum time or maximum profit.

Let us understand basic difference between transportation and assignment problem.

Transportation Problem	Assignment Problem
This problem contains specific demand and requirement in columns and rows	The demand and requirement in each column or row is one
The no. of rows may not be equal to the no. of columns.	It is a square matrix. The no. of rows must be equal to the no. of columns.
There is no restriction in the number of allotments in any row or column	There should be only one allotment in each row and each column

2. General Form of Assignment Problem

Consider a general assignment problem consisting of n origins (sources) O_1, $O_2 \ldots O_n$ and n destinations (sinks) D_1, $D_2 \ldots D_n$. Let the cost or time for associating i^{th} origin to j^{th} destination is c_{ij}.

The cost matrix can be shown as follows:

	D_1	D_2	$\cdots$	D_n	**Supply**

O_1	c_{11}	c_{12}		c_{1n}	**1**
O_2	c_{21}	c_{22}		c_{2n}	**1**
$\vdots$					
O_n	c_{n1}	c_{n2}		c_{nn}	**1**
Requirement	1	1		1	

Let the amount of commodity supplied from *i* to *j* be denoted as x_{ij}.

Here x_{ij} is defined as

$x_{ij} = 1$, if the i^{th} origin is associated to the j^{th} destination

$x_{ij} = 0$, if the i^{th} origin is not associated to the j^{th} destination

Note:

If the number of rows is equal to the number of columns then it is called a **balanced assignment problem**.

But if the number of rows is not equal to the number of columns then it is called an **unbalanced assignment problem**.

3. The Hungarian Method

The following steps are followed to a given n × n cost matrix to find an optimal assignment.

1. Subtract the smallest element in each row from all the elements of its row.

2. Subtract the smallest element in each column from all the elements of its column.

3. Draw lines through appropriate rows and columns so that all the zero elements of the cost matrix are covered and the minimum number of such lines is used.

4. Tests for Optimality: (i) If the minimum number of covering lines is n, an optimal assignment of zeros is possible and we are finished. (ii) If the minimum number of covering lines is less

than n, an optimal assignment of zeros is not yet possible. In that case, proceed to Step 5.

5. Determine the smallest element not covered by any line. Subtract this element from each uncovered elements, and then add it to each elements where lines intersect. Return to Step 3

4. Example 1 with Solution

Example 1: Suppose you work as a sales manager for a toy manufacturer, and you currently have three salespersons located in different cities A, B and C. You want them to travel to three other cities: D, E and F. The table below shows the cost (Rs.) of bus tickets between cities they are located and the cities they have to travel.

	D	**E**	**F**
A	250	400	350
B	400	600	350
C	200	400	250

Where should you send each of your salespersons in order to minimize bus ticket fare?

Solution: We shall solve this problem using Hungarian method:

Step1: Subtract minimal element from all elements of that row.

	D	**E**	**F**
A	250	400	350
B	400	600	350
C	200	400	250

Subtract 250 from all elements of first row, 350 from all elements of second row and 200 from all elements of third row.

	D	**E**	**F**
A	250-250=0	400-250=150	350-250=100
B	400-350=50	600-350=250	350-350=0
C	200-200=0	400-200=200	250-200=50

Step2: Subtract minimal element from all elements of that column in the resultant matrix.

	D	E	F
A	0	150	100
B	50	250	0
C	0	200	50

Subtract 0 from all elements of first column, 150 from all elements of second column and 0 from all elements of the third column.

	D	E	F
A	0-0=0	150-150=0	100-0=100
B	50-0=50	250-150=100	0-0=0
C	0-0=0	200-150=50	50-0=50

Step3: Cover all the zeros of the matrix with the minimum number of horizontal or vertical lines

	D	E	F
A	0	0	100
B	50	100	0
C	0	50	50

Number of minimum number lines = No. of rows, optimal solution can be obtained.

Step4: Assign to zeroes

	D	E	F
A	0	0	100
B	50	100	0
C	0	50	50

	D	E	F
A	0	0	100
B	50	100	0
C	0	50	50

	D	E	F
A	0	0	100
B	50	100	0
C	0	50	50

	D	E	F
A	0	0	100
B	50	100	0
C	0	50	50

Here is the same assignment made to the original cost matrix.

	D	E	F
A	250	400	350
B	400	600	350
C	200	400	250

Thus final allocation and corresponding allocation cost is tabulated below:

Assignment	**Assignment Cost (Rs.)**
Salesman from city A to city E	400
Salesman from city B to city F	350
Salesman from city C to city D	200
Total allocation cost	**950**

5. Example 2 with Solution

Example2: In a factory 3 jobs are to be assigned to 3 machines. The time (in hrs) taken by machine to complete the assigned job is given by the below matrix. Assign the jobs to the machines in such a way that total time taken is minimum.

Jobs	Machines		
	I	II	III
1	19	28	31
2	11	17	16
3	12	15	13

Solution: We shall solve this problem using Hungarian method:

Step1: Subtract minimal element from all elements of that row.

Jobs	Machines		
	I	II	III
1	19	28	31
2	11	17	16
3	12	15	13

Subtract 19 from all elements of first row, 11 from all elements of second row and 12 from all elements from third row.

Jobs	Machines		
	I	II	III
1	19-19=0	28-19=9	31-19=12
2	11-11=0	17-11=6	16-11=5
3	12-12=0	15-12=3	13-12=1

Step2: Subtract minimal element from all elements of that column in the resultant matrix.

Jobs	Machines		
	I	II	III
1	0	9	12
2	0	6	5
3	0	3	1

Subtract 0 from all elements of first column, 3 from all elements of second column and 1 from all elements of third column.

Jobs	Machines		
	I	II	III

1	0-0=0	9-3=6	12-1=11
2	0-0=0	6-3=3	5-1=4
3	0-0=0	3-3=0	1-1=0

Step3: Cover all the zeros of the matrix with the minimum number of horizontal or vertical lines.

Jobs	Machines		
	I	II	III
1	0	6	11
2	0	3	4
3	0	0	0

Number of lines = 2, Number of rows = 3

As number of lines < number of rows, optimal assignment of zeroes is yet not possible.

The smallest element not covered by any line is 3. Subtract 3 from each uncovered elements, and then add it to each elements where lines intersect.

Jobs	Machines		
	I	II	III
1	0	3	8
2	0	0	1
3	3	0	0

We shall again cover all zeroes with minimum number of lines.

Jobs	Machines		
	I	II	III
1	0	3	8
2	0	0	1
3	3	0	0

Number of lines = 3, Number of rows = 3

As number of lines = number of rows, optimal assignment of zeroes is now possible.

Step4: Assign to zeroes.

Jobs	Machines		
	I	II	III
1	0	3	8

2	0	0	1
3	3	0	0

Here is the same assignment made to the original cost matrix.

Jobs	Machines		
	I	II	III
1	19	28	31
2	11	17	16
3	12	15	13

Thus final allocation and corresponding time is tabulated below:

Assignment	**Time (hrs.)**
Job 1 to Machine I	19
Job 2 to Machine II	17
Job 3 to Machine III	13
Total allocation time	**49**

6. Example 3 with Solution

Example3: In a workshop there are 4 mechanics and 5 jobs. Time taken to complete each job is given below. Solve this assignment problem.

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	9	6	5	4	2
B	7	6	3	2	8
C	6	7	4	5	3
D	2	6	4	9	6

Solution: In this assignment problem there are 4 rows and 5 columns. So, it is an unbalanced assignment problem. Here there is one less number of row compared to number of column. To convert it into a balanced assignment problem, we shall add a dummy row with zero cost.

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	9	6	5	4	2
B	7	6	3	2	8
C	6	7	4	5	3
D	2	6	4	9	6
Dummy	0	0	0	0	0

We shall solve this problem using Hungarian method:

Step1: Subtract minimal element from all elements of that row.

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	9	6	5	4	2
B	7	6	3	2	8
C	6	7	4	5	3
D	2	6	4	9	6
Dummy	0	0	0	0	0

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	9-2=7	6-2=4	5-2=3	4-2=2	2-2=0
B	7-2=5	6-2=4	3-2=1	2-2=0	8-2=6
C	6-3=3	7-3=4	4-3=1	5-3=2	3-3=0
D	2-2=0	6-2=4	4-2=2	9-2=7	6-2=4
Dummy	0-0=0	0-0=0	0-0=0	0-0=0	0-0=0

Step2: Subtract minimal element from all elements of that column in the resultant matrix.

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	7	4	3	2	0
B	5	4	1	0	6
C	3	4	1	2	0
D	0	4	2	7	4
Dummy	0	0	0	0	0

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**

A	7-0=7	4-0=4	3-0=3	2-0=2	0-0=0
B	5-0=5	4-0=4	1-0=1	0-0=0	6-0=6
C	3-0=3	4-0=4	1-0=1	2-0=2	0-0=0
D	0-0=0	4-0=4	2-0=2	7-0=7	4-0=4
Dummy	0-0=0	0-0=0	0-0=0	0-0=0	0-0=0

Step3: Cover all the zeros of the matrix with the minimum number of horizontal or vertical lines.

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	7	4	3	2	0
B	5	4	1	0	6
C	3	4	1	2	0
D	0	4	2	7	4
Dummy	0	0	0	0	0

Number of lines = 4, Number of rows = 5

As number of lines < number of rows, optimal assignment of zeroes is yet not possible.

The smallest element not covered by any line is 1. Subtract 1 from each uncovered elements, and then add it to each elements where lines intersect.

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	6	3	2	1	0
B	5	4	1	0	7
C	2	3	0	1	0
D	0	4	2	7	5
Dummy	0	0	0	0	1

We shall again cover all zeroes with minimum number of lines.

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	6	3	2	1	0
B	5	4	1	0	7
C	2	3	0	1	0
D	0	4	2	7	5
Dummy	0	0	0	0	1

Number of lines = 5, Number of rows = 5

As number of lines = number of rows, optimal assignment of zeroes is now possible.

Step4: Assign to zeroes

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	6	3	2	1	[0]
B	5	4	1	[0]	7
C	2	3	[0]	1	~~0~~
D	[0]	4	2	7	5
Dummy	~~0~~	[0]	~~0~~	~~0~~	1

Here is the same assignment made to the original cost matrix.

Mechanics	Jobs				
	1	**2**	**3**	**4**	**5**
A	9	6	5	4	[2]
B	7	6	3	[2]	8
C	6	7	[4]	5	3
D	[2]	6	4	9	6
Dummy	0	[0]	0	0	0

Thus final allocation and corresponding time is tabulated below:

Assignment	**Time (hrs.)**
Mechanic A to Job 5	2
Mechanic B to Job 4	2
Mechanic C to Job 3	4

Mechanic D to Job 1	2
Total allocation cost	**10**

Here is important note that Job 2 is assigned to dummy mechanic, so Job2 will remain unattended.

7. Summary

- The assignment problem is the special case of the transportation problem in which the objective is to assign a number of origins to the equal number of destinations at a minimum cost, minimum time or maximum profit.
- It is a square matrix. The no. of rows must be equal to the no. of columns.
- The demand and requirement in each column or row is one.
- There should be only one allotment in each row and each column
- **General form of A.P.**

  The cost matrix can be shown as follows:

	D1	**D2**	⋯	**Dn**	Supply
O1	c_{11}	c_{12}		c_{1n}	1
O2	c_{21}	c_{22}		c_{2n}	1
⋮					
On	c_{n1}	c_{n2}		c_{nn}	1
Requirement	1	1		1	

- Here x_{ij} is the amount of commodity supplied from *i* to *j* and defined as

  $x_{ij} = 1$, if the *ith* origin is associated to the *jth* destination

  $x_{ij} = 0$, if the *ith* origin is not associated to the *jth* destination

- If the number of rows is equal to the number of columns then it is called a **balanced assignment problem**.
- But if the number of rows is not equal to the number of columns then it is called an **unbalanced assignment problem**.
- Unbalanced problem are converted into balanced assignment problem by adding a dummy row or column.