[Academic Script]
[Transportation Problem (Part - 2)]

Subject: Business Economics

Course: B.A., 4th Semester, Undergraduate

Paper No. & Title: Paper – 403 (Four Zero Three) International Economics

Unit No. & Title: Unit – 2(two) Transportation & Assignment

Lecture No. & Title: 2(Two): Transportation Problem (Part - 2)

Transportation Problem - 2

Introduction

In the previous lecture, we have seen what is transportation problem and discussed about the standard form of a transportation problem. We studied methods namely North West corner, matrix minima and Vogel's for obtaining initial basic feasible solution.
In this lecture we shall check whether the obtained initial basic feasible solution is optimal or not? If it is not optimal we shall obtain optimal solution using stepping stone method or by MODI method.

Test for Optimality

Optimality test can be performed if two conditions are satisfied i.e.

1. There are m + n – 1 allocations, where m is number of rows, n is number of columns.

2. There are m + n – 1 allocations should be at independent positions. i.e. it should not be possible to increase or decrease any allocation without either changing the position of the allocations or violating the row or column restrictions.

A simple rule for allocations to be in independent positions is that it is impossible to travel from any allocation, back to itself by a series of horizontal or vertical steps from one occupied cell to another, without a direct reversal of route.

Optimal Solution

Following two methods are commonly used methods to obtain optimal solution of the transportation problem.

1. Stepping Stone Method 2. MODI method

1. Stepping Stone Method

The stepping-stone method will help us move from an initial feasible solution to an optimal solution. It is used to evaluate the cost effectiveness of transporting goods via transportation routes not currently in the solution. When applying it, we test each unallocated cell in the transportation table by asking: What would happen to total transportation costs if one unit of the product was tentatively transported to an unallocated route?

STEPS

1. Select any unallocated cell to evaluate.
2. Beginning with this cell, trace a closed path back to the original cell via cells that are currently being allocated (only horizontal and vertical moves are permissible). You may, however, step over either an unallocated or allocated cells.
3. Beginning with a plus (+) sign at the unallocated cells, place alternating minus signs and plus signs on each corner cells of the closed path just traced.
4. Calculate an improvement index by first adding the unit-cost figures found in each cell containing a plus sign and then by subtracting the unit costs in each cell containing a minus sign.
5. Repeat steps 1 through 4 until you have calculated an improvement index for all unallocated cells. If all indices computed are greater than or equal to zero, you have reached an optimal solution. If not, the current solution can be improved further to decrease total shipping costs.

Example 1: Obtain the initial basic feasible solution by least cost method. Check using the stepping stone method whether i. b. f. s. is optimal solution.

	D1	D2	D3	D4	Supply
S1	13	17	16	14	50
S2	12	14	13	12	25
S3	14	13	18	15	25
Demand	30	30	20	20	

Solution: The initial basic feasible solution using least cost method is as follows:

	D1	D2	D3	D4	Supply
S1	[5] 13	[5] 17	[20] 16	[20] 14	50
S2	[25] 12	14	13	12	25
S3	14	[25] 13	18	15	25
Demand	30	30	20	20	

Total cost= 5*13 + 5*17 + 20*16 + 20*14 + 25*12 + 25*13
= 1375 Rs.

In the above initial basic feasible solution table, cells S2D2, S2D3, S2D4, S3D1, S3D3, S3D4 are unallocated.

Let us start allocation with cell S2D2 to assess whether there is improvement in the solution

	D1	D2	D3	D4	Supply
S1	[5] + 13	[5] - 17	[20] 16	[20] 14	50
S2	[25] 12 -	14 +	13	12	25
S3	14	[25] 13	18	15	25
Demand	30	30	20	20	

Net change in transportation cost: +14 – 12 +13 – 17 = -2.

Similar procedure is to be done for all other unallocated cells.Net change in transportation cost after allocating to the unallocated cells will be as shown:

Allocation	Net change in transportation cost
S2D2	+14 -12+ 13-17 = -2
S2D3	+13 –12+13 -16 = -2
S2D4	+12- 12+ 13-14 = -1
S3D1	+14 -13+17-13 = +5
S3D3	+18 -13+17-16 = +6
S3D4	+15 -13+17-14 = +5

As net changes are negative for cells S2D2, S2D3 and S2D4, the obtained initial basic feasible solution is not optimal.
We shall now select the loop with the most negative value of net change in transportation cost. It is -2 for cell S2D2 and S2D3. Arbitrarily selecting cell S2D2.

	D1	D2	D3	D4	Supply
S1	[5] 13 +5	[5] 17 -5	[20] 16	[20] 14	50
S2	[25] 12 -5	14 +5	13	12	25
S3	14	[25] 13	18	15	25
Demand	30	30	20	20	

To further improve the solution, the minimum element in the loop with minus sign is selected. Here it is 5. Adding this 5 to elements with plus sign and subtracting it from elements with minus sign.

	D1	D2	D3	D4	Supply
S1	10 13	17	20 16	20 14	50
S2	20 12	5 14	13	12	25
S3	14	25 13	18	15	25
Demand	30	30	20	20	

Again the net changes in transportation cost are to be calculated for all unallocated cells. The above process continues till net changes in transportation cost for all unallocated cells are positive.

2. **Modified Distribution Method (MODI)**

It is a method for computing optimum solution of a transportation problem.

STEPS

1. Determine an initial basic feasible solution using any one of the three methods given below:
    - North West Corner Rule
    - Matrix Minimum Method
    - Vogel Approximation Method

2. Determine the values of dual variables, $\overline{u_i}$ and $\overline{v_j}$, using $\overline{u_i} + \overline{v_j} = \overline{c_{ij}}$ for allocated cells.
3. Compute the opportunity cost for all unallocated cells using $\overline{c_{ij}} - (\overline{u_i} + \overline{v_j})$.
4. Check the sign of each opportunity cost. If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimum solution. On the other hand, if one or more unoccupied cell has negative opportunity cost, the given solution is not an optimum solution and further savings in transportation cost are possible.
5. Select the unoccupied cell with the smallest negative opportunity cost as the cell to be included in the next solution.
6. Draw a closed path or loop for the unoccupied cell selected in the previous step. Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cell.
7. Assign alternate plus and minus signs at the unoccupied cells on the corner points of the closed path with a plus sign at the cell being evaluated.
8. Determine the maximum number of units that should be shipped to this unoccupied cell. The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell. Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs and subtract it from those cells marked with minus signs. In this way an unoccupied cell becomes an occupied cell.
9. Repeat the whole procedure until an optimum solution is obtained.

Example 2: Obtain initial basic feasible solution by Vogel's method. Obtain optimal solution using MODI method.

Origins	Destinations						Availability
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	
O1	1	2	1	4	5	2	30
O2	3	3	2	1	4	3	50
O3	4	2	5	9	6	2	75
O4	3	1	7	3	4	6	20
Requirements	20	40	30	10	50	25	

Solution:

Using Vogel's method, an initial basic feasible solution is given in following table:

Origins	Destinations					
	D1	**D2**	**D3**	**D4**	**D5**	**D6**
O1	[20] 1	2	[10] 1	4	5	2
O2	3	3	[20] 2	[10] 1	[20] 4	3
O3	4	[20] 2	5	9	[30] 6	[25] 2
O4	3	[20] 1	7	3	4	6

Total transportation cost = 20*1 + 10*1 + 20*2 + 10*1 + 20*4 + 20*2 + 30*6 + 25*2 + 20*1

= 450 Rs.

We shall obtain optimal solution using MODI method.

We shall find the values of dual variables, $\overline{u_i}$ and $\overline{v_j}$, using $\overline{u_i}$ + $\overline{v_j}$ = $\overline{c_{ij}}$ for allocated cells and also compute the opportunity cost for all unallocated cells using $\overline{c_{ij}}$ – ($\overline{u_i}$ + $\overline{v_j}$).

Starting by taking $\overline{u_2}$=0.

$\overline{u_2 + v_3}$= 2 $\therefore \overline{0 + v_3}$= 2 $\therefore$ $\overline{v_3}$=2

$\overline{u_2 + v_4}$= 1 $\therefore \overline{0 + v_4}$= 1 $\therefore$ $\overline{v_4}$=1

$\overline{u_2 + v_5}$= 4 $\therefore \overline{0 + v_5 = 4}$ $\therefore v_5$=4

Now,

$\overline{v_3 + u_1}$= 1 $\therefore \overline{2 + u_1 = 1}$ $\therefore u_1$=-1

$\overline{v_5 + u_3}$= 6 $\therefore \overline{4 + u_3 = 6}$ $\therefore u_3$=2

$\overline{u_1 + v_1}$= 1 $\therefore \overline{-1 + v_1 = 1}$ $\therefore$ v_1=2

$\overline{u_3 + v_2}$= 2 $\therefore \overline{2 + v_2 = 2}$ $\therefore$ v_2=0

$\overline{u_3 + v_6}$= 2 $\therefore \overline{2 + v_6 = 2}$ $\therefore$ v_6=0

$\overline{v_2 + u_4}$= 1 $\therefore \overline{0 + u_4 = 1}$ $\therefore u_4$=1

Putting this values of dual variables $\overline{u_i}$ and $\overline{v_j}$, we get table as shown

Origins	Destinations						$\overline{u_i}$
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	
O1	[20] 1	2	[10] 1	4	5	2	-1
O2	3	3	[20] 2	[10] 1	[20] 4	3	0
O3	4	[20] 2	5	9	[30] 6	[25] 2	2
O4	3	[20] 1	7	3	4	6	1
$\overline{v_j}$	2	0	2	1	4	0	

We shall find opportunity cost for all unallocated cells using $\overline{c_{ij}}$ – ($\overline{u_i}$ + $\overline{v_j}$).

Origins	Destinations						$\overline{u_i}$
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	
O1	[20] 1	(3) 2	[10] 1	(4) 4	(2) 5	(3) 2	-1
O2	(1) 3	(3) 3	[20] 2	[10] 1	[20] 4	(3) 3	0
O3	(0) 4	[20] 2	(1) 5	(6) 9	[30] 6	[25] 2	2
O4	(1) 3	[20] 1	(4) 7	(1) 3	(-1) 4	(5) 6	1
$\overline{v_j}$	2	0	2	1	4	0	

Now as one the value of $\overline{c_{ij}}$ – ($\overline{u_i}$ + $\overline{v_j}$) i.e $\overline{c_{45}}$ – ($\overline{u_4}$ + $\overline{v_5}$) is negative, the solution is not optimal. We shall make a loop starting from cell O4D5.

Origins	Destinations						$\overline{u_i}$
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	
O1	[20] 1	(3) 2	[10] 1	(4) 4	(2) 5	(3) 2	*-1*
O2	(1) 3	(3) 3	[20] 2	[10] 1	[20] 4	(3) 3	*0*
O3	(0) 4	[20] 2 +	(1) 5	(6) 9	[30] 6 -	[25] 2	*2*
O4	(1) 3	[20] 1 -	(4) 7	(1) 3	(-1) 4 +	(5) 6	*1*
$\overline{v_j}$	*2*	*0*	*2*	*1*	*4*	*0*	

Cell O4D5 will be allocated 20 units and accordingly reallocation will be done in the allocated cells of the loop. And the new table will be as follows:

Origins	Destinations					
	D1	**D2**	**D3**	**D4**	**D5**	**D6**
O1	[20] 1	2	[10] 1	4	5	2
O2	3	3	[20] 2	[10] 1	[20] 4	3
O3	4	[40] 2	5	9	[10] 6	[25] 2
O4	3	1	7	3	[20] 4	6

We shall again find the values of dual variables and also find opportunity cost for all unallocated cells.

Origins	Destinations						$\overline{u_i}$
	D1	**D2**	**D3**	**D4**	**D5**	**D6**	
O1	[20] 1	(3) 2	[10] 1	(4) 4	(2) 5	(3) 2	*-1*
O2	(1) 3	(3) 3	[20] 2	[10] 1	[20] 4	(3) 3	*0*
O3	(0) 4	[40] 2	(1) 5	(6) 9	[10] 6	[25] 2	*2*
O4	(1) 3	(1) 1	(5) 7	(2) 3	[20] 4	(6) 6	*0*
$\overline{v_j}$	*2*	*0*	*2*	*1*	*4*	*0*	

Now as all the values of $\overline{c_{ij}} - (\overline{u_i} + \overline{v_j})$ are non-negative, the solution is optimal.

The minimal transportation cost is 20*1 + 10*1 + 20*2 + 10*1 + 20*4 + 40*2 + 10*6 + 25*2 + 20*4 = 430 Rs.

SUMMARY

- Methods namely North West corner, Matrix minima and Vogel's are used for obtaining initial basic feasible solution.

- **Test for Optimality**

Optimality test can be performed if two conditions are satisfied i.e.

1. There are m + n – 1 allocations.

2. These m + n – 1 allocations should be at independent positions.

- Two methods commonly used to obtain optimal solution of the transportation problem are
  1. Stepping Stone Method   2. MODI method

- The **stepping-stone method** will help us move from an initial feasible solution to an optimal solution. It is used to evaluate the cost effectiveness of transporting goods via transportation routes not currently in the solution.
- The Modified Distribution Method **(MODI)** is a method of computing optimum solution of a transportation problem.
- In this method the sign of each opportunity cost is checked. If the opportunity costs of all the unoccupied cells are either positive or zero, the given solution is the optimum solution. On the other hand, if one or more unoccupied cell has negative opportunity cost, the given solution is not an optimum solution and further savings in transportation cost are possible.