[Academic Script]

Transportation Problem (Part - 1)

Subject:	Business Economics
Course:	B.A., 4th Semester, Undergraduate
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1. Introduction

In the previous lectures, we discussed about the standard form of a Linear Programming and the commonly used methods of solving Linear Programming Problem. A key problem in many projects is the allocation of limited resources among various activities. Transportation problem refers to a planning model that allocates resources, machines, materials, capital etc. in a best possible way so that the costs are minimized or profits are maximized. In this lecture, the common structure of a transportation problem (TP) and its solution using different method are discussed followed by a numerical example.

The classic transportation problem is concerned with the distribution of any commodity (resource) from any group of 'sources' to any group of destinations or 'sinks'. While solving this problem using LP, the amount of resources from source to sink will be the decision variables. The criterion for selecting the optimal values of the decision variables (like minimization of costs or maximization of profits) will be the objective function. And the limitation of resource availability from sources will constitute the constraint set.

2. General form of Transportation Problem

Consider a general transportation problem consisting of m origins (sources) O1, O2,…, Om and n destinations (sinks) D1, D2, … , Dn. Let the amount of commodity available in i th source be a_i(i=1,2,….m) and the demand in j th sink be b_j (j=1,2,….n). Let the cost of transportation of unit amount of material from i to j be c_{ij}. The transportation cost matrix can be shown as follows:

	D1	**D2**	…	**Dn**	Supply
O1	c_{11}	c_{12}		c_{1n}	$\boldsymbol{a_1}$
O2	c_{21}	c_{22}		c_{2n}	$\boldsymbol{a_2}$
⋮					
Om	c_{m1}	c_{m2}		c_{mn}	$\boldsymbol{a_m}$
Demand	$\boldsymbol{b_1}$	$\boldsymbol{b_2}$		$\boldsymbol{b_n}$	$\sum a_i = \sum b_j$

Let the amount of commodity supplied from *i* to *j* be denoted as x_{ij}. Thus, the cost of transporting x_{ij} units of commodity from *i* to *j* is $c_{ij} \times x_{ij}$.

Note:

If the total supply is equal to the total demand then it is called a **balanced transportation problem**.

But if the total supply is not equal to the total demand then it is called an **unbalanced transportation problem**.

Methods of solving T.P.

Following three methods are commonly used methods to obtain initial basic feasible solution of the transportation problem.

1. North West Corner method
2. Least Cost method (or Matrix minima method)
3. Vogel's method

3. North West Corner Method

The north-west corner method generates an initial allocation according to the following procedure:

- Allocate the maximum amount allowable by the supply and demand constraints to the variable x_{11} (i.e. the cell in the top left corner of the transportation table).

- If a column (or row) is satisfied, cross it out. The remaining decision variables in that column (or row) are non-basic and are set equal to zero. If a row and column are satisfied simultaneously, cross only one out (it does not matter which).
- Adjust supply and demand for the non-crossed out rows and columns.
- Allocate the maximum feasible amount to the first available non-crossed out element in the next column (or row).
- When exactly one row or column is left, all the remaining variables are basic and are assigned the only feasible allocation.

Example1. Solve the following transportation problem using North West Corner method. Also obtain the total transportation cost.

	Selling Centers			
Production Plants	P	Q	R	Supply
A	8	5	6	120
B	15	10	12	80
C	3	9	10	80
Demand	150	70	60	

Solution:

	Selling Centers			
Production Plants	P	Q	R	Supply
A	120 8	5	6	~~120~~ 0
B	15	10	12	80
C	3	9	10	80
Demand	~~150~~	70	60	

	30			

	Selling Centers			
Production Plants	P	Q	R	Supply
A	120 8	5	6	~~120~~ 0
B	30 15	10	12	~~80~~ 50
C	3	9	10	80
Demand	~~150~~ ~~30~~ 0	70	60	

	Selling Centers			
Production Plants	P	Q	R	Supply
A	120 8	5	6	~~120~~ 0
B	30 15	50 10	12	~~80~~ ~~50~~ 0
C	3	9	10	80
Demand	~~150~~ ~~30~~ 0	~~70~~ 20	60	

	Selling Centers			
Production Plants	P	Q	R	Supply
A	120 8	5	6	~~120~~ 0
B	30 15	50 10	12	~~80~~ ~~50~~ 0
C	3	20 9	10	~~80~~ 60
Demand	~~150~~ ~~30~~ 0	~~70~~ ~~20~~ 0	60	

	Selling Centers			
Production Plants	P	Q	R	Supply
A	120 8	5	6	~~120~~ 0
B	30 15	50 10	12	~~80~~ ~~50~~ 0
C	3	20 9	60 10	~~80~~ ~~60~~ 0
Demand	~~150~~ ~~30~~ 0	~~70~~ ~~20~~ 0	~~60~~ 0	

Thus allocation of units and total cost of transportation is:

Allocation	**Units**	**Cost**
A to P	120	120*8 = 960
B to P	30	30*15 = 450
B to Q	50	50*10 = 500
C to Q	20	20*9 = 180
C to R	60	60*10 = 600
	Total Cost	Rs. 2690

4. The Least-Cost Method

This method usually provides a better initial basic feasible solution than the North-West Corner method since it takes into account the cost variables in the problem. Procedure used for obtaining the solution is as follows:

- Assign as much as possible to the cell with the smallest unit cost in the entire table. If there is a tie then choose arbitrarily.
- Cross out the row or column which has satisfied supply or demand. If a row and column are both satisfied then cross out only one of them.

- Adjust the supply and demand for those rows and columns which are not crossed out.
- When exactly one row or column is left, all the remaining variables are basic and are assigned the only feasible allocation.

Example2. Solve the following transportation problem using Least Cost method. Also obtain total transportation cost.

	Destination			
Origins	D1	D2	D3	Supply
O1	3	4	5	100
O2	5	2	6	120
O3	1	6	3	130
O4	5	4	4	50
Demand	150	80	170	

Solution:

	Destination			
Origins	D1	D2	D3	Supply
O1	3	4	5	100
O2	5	2	6	120
O3	13 1	6	3	~~130~~ 0
O4	5	4	4	50
Demand	~~150~~ 20	80	170	

	Destination			
Origins	D1	D2	D3	Supply

O1	3	4	5	100
O2	5	80 2	6	~~120~~ 40
O3	13 1	6	3	~~130~~ 0
O4	5	4	4	50
Demand	~~150~~ 20	~~80~~ 0	170	

	Destination			
Origins	D1	D2	D3	Supply
O1	20 3	4	5	~~100~~ 80
O2	5	80 2	6	~~120~~ 40
O3	13 1	6	3	~~130~~ 0
O4	5	4	4	50
Demand	~~150~~ ~~20~~ 0	~~80~~ 0	170	

	Destination			
Origins	D1	D2	D3	Supply
O1	20 3	4	5	~~100~~ 80
O2	5	80 2	6	~~120~~ 40
O3	13 1	6	3	~~130~~ 0
O4	5	4	50 4	~~50~~ 0
Demand	~~150~~	~~80~~	~~170~~	

	~~20~~ 0	0	120	

	Destination			
Origins	D1	D2	D3	Supply
O1	20 3	4	80 5	~~100~~ ~~80~~ 0
O2	5	80 2	6	~~120~~ 40
O3	13 1	6	3	~~130~~ 0
O4	5	4	50 4	~~50~~ 0
Demand	~~150~~ ~~20~~ 0	~~80~~ 0	~~170~~ ~~120~~ 40	

	Destination			
Origins	D1	D2	D3	Supply
O1	20 3	4	80 5	~~100~~ ~~80~~ 0
O2	5	80 2	40 6	~~120~~ ~~40~~ 0
O3	13 1	6	3	~~130~~ 0
O4	5	4	50 4	~~50~~ 0
Demand	~~150~~ ~~20~~ 0	~~80~~ 0	~~170~~ ~~120~~ ~~40~~ 0	

Thus allocation of units and total cost of transportation is:

Allocation	**Units**	**Cost**
O1to D1	20	20*3 = 60
O1 to D3	80	80*5 = 400
O2 to D2	80	80*2 = 160
O3 to D1	130	130*1 = 130

O4 to D3	50	50*4 = 200
	Total Cost	Rs. 950

5. Vogel's Approximation Method of Allocation

This method also takes costs into account in allocation. Following procedure is used to obtain the solution:

- Determine the difference between the lowest two cells in all rows and columns, including dummies.
- Identify the row or column with the largest difference. Ties may be broken arbitrarily.
- Allocate as much as possible to the lowest-cost cell in the row or column with the highest difference. If two or more differences are equal, allocate as much as possible to the lowest-cost cell in these rows or columns.
- Stop the process if all row and column requirements are met. If not, go to the next step.
- Recalculate the differences between the two lowest cells remaining in all rows and columns. Any row and column with zero supply or demand should not be used in calculating further differences.

Example3. Solve the following transportation problem using Vogel's method. Also obtain total transportation cost.

	Shops				
Warehouse	**A1**	**A2**	**A3**	**A4**	Supply
1	5	4	3	6	1000
2	8	4	3	5	800
3	9	7	5	4	700
Demand	500	900	600	500	

Solution:

	Shops					
Warehouse	**A1**	**A2**	**A3**	**A4**	Supply	Diff1
1	5	4	3	6	1000	1
2	8	4	3	5	800	1
3	9	7	5	4	700	1
Demand	500	900	600	500		
Diff1	3	0	0	1		

	Shops					
Warehouse	**A1**	**A2**	**A3**	**A4**	Supply	Diff1
1	500 5	4	3	6	~~1000~~ 500	1
2	8	4	3	5	800	1
3	9	7	5	4	700	1
Demand	~~500~~ 0	900	600	500		
Diff1	3	0	0	1		

	Shops						
Warehouse	**A1**	**A2**	**A3**	**A4**	Supply	Diff1	Diff2
1	500 5	4	3	6	~~1000~~ 500	1	1
2	8	4	3	5	800	1	1
3	9	7	5	4	700	1	1
Demand	~~500~~	900	600	500			

	0						
Diff1	3 ↑	0	0	1			
Diff2	---	0	0	1			

	Shops						
Warehouse	**A1**	**A2**	**A3**	**A4**	Supply	Diff1	Diff2
1	500 5	4	3	6	~~1000~~ 500	1	1
2	8	4	600 3	5	~~800~~ 200	1	1 ←
3	9	7	5	4	700	1	1
Demand	~~500~~ 0	900	~~600~~ 0	500			
Diff1	3 ↑	0	0	1			
Diff2	---	0	0	1			

	Shops							
Ware-house	**A1**	**A2**	**A3**	**A4**	Supply	Diff1	Diff2	Diff3
1	500 5	4	3	6	~~1000~~ 500	1	1	2
2	8	4	600 3	5	~~800~~ 200	1	1 ←	1
3	9	7	5	4	700	1	1	3 ←
Demand	~~500~~ 0	900	~~600~~ 0	500				
Diff1	3 ↑	0	0	1				
Diff2	---	0	0	1				
Diff3	---	0	---	1				

	Shops							
Ware-	**A1**	**A2**	**A3**	**A4**	Supply	Diff1	Diff2	Diff3

house								
1	[500] 5	4	3	6	~~1000~~ 500	1	1	2
2	8	4	[600] 3	5	~~800~~ 200	1	1 ←	1
3	9	7	5	[500] 4	~~700~~ 200	1	1	3 ←
Demand	~~500~~ 0	900	~~600~~ 0	~~500~~ 0				
Diff1	3 ↑	0	0	1				
Diff2	---	0	0	1				
Diff3	---	0	---	1				

As only one column is left, allocation is made to all unallocated elements of that column. Here it is second column.

	Shops							
Ware-house	**A1**	**A2**	**A3**	**A4**	Supply	Diff1	Diff2	Diff3
1	[500] 5	[500] 4	3	6	~~1000~~ ~~500~~ 0	1	1	2
2	8	[200] 4	[600] 3	5	~~800~~ ~~200~~ 0	1	1 ←	1
3	9	[200] 7	5	[500] 4	~~700~~ ~~200~~ 0	1	1	3 ←
Demand	~~500~~ 0	~~900~~ 0	~~600~~ 0	~~500~~ 0				
Diff1	3 ↑	0	0	1				
Diff2	---	0	0	1				
Diff3	---	0	---	1				

Thus allocation of units and total cost of transportation is:

Allocation	**Units**	**Cost**
1 to A1	500	500*5 =2500
1 to A2	500	500*4 =2000
2 to A2	200	200*4 =0800

2 to A3	600	600*3 =1800
3 to A2	200	200*7 =1400
3 to A4	500	500*4 =2000
	Total Cost	Rs. 10500

6. Summary

- Transportation problem refers to a planning model that allocates resources, machines, materials, capital etc. in a best possible way so that the costs are minimized or profits are maximized.
- The general form of transportation problem is:

	D1	**D2**	···	**Dn**	Supply
O1	c_{11}	c_{12}		c_{1n}	$\boldsymbol{a_1}$
O2	c_{21}	c_{22}		c_{2n}	$\boldsymbol{a_2}$
⋮					
Om	c_{m1}	c_{m2}		c_{mn}	$\boldsymbol{a_m}$
Demand	$\boldsymbol{b_1}$	$\boldsymbol{b_2}$		$\boldsymbol{b_n}$	$\sum a_i = \sum b_j$

- If the total supply is not equal to the total demand then it is called an **unbalanced transportation problem**.
- Three methods commonly used to obtain initial basic feasible solution of the transportation problem are.

1. North West Corner method
2. Least Cost method (or Matrix minima method)
3. Vogel's method

- In North West Corner method the first step is to allocate maximum amount allowable by the supply and demand constraints to the cell in the top left corner of the transportation table.
- In Least Cost method the first step is to allocate maximum amount allowable by the supply and demand constraints to the minimum value of the matrix.

- In Vogel’s method the first step is to allocate maximum amount allowable by the supply and demand constraints to the minimum cost of the highest difference row or column.