[Academic Script]

Quantitative Techniques for Management

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Lecture No. & Title:	3: Basic Inventory Models

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Basic Inventory Models

1. Introduction:

Inventory means raw materials used in process, finished products, packaging, and offers – stocked in order to meet an expected demand in the future. Thus inventory consists of idle resources. The resources may be of any type – materials, machines, goods, men, etc.

Inventory control is the process of deciding what and how much of various items are to be kept in stock. The basic object of inventory control is to reduce investment in the inventories, ensuring that the production process does not suffer at the same time.

2. Cost associated with inventories

1. Set up cost: The cost associated with setting up of machinery before starting production.
2. Ordering cost: The cost of ordering raw material for production purposes. It also includes the cost of advertisements, telephone charges, rent for space used by the purchasing department, travelling expenditure, etc.
3. Purchase (Production) cost: The cost of purchasing (or producing) a unit of an item is known as purchase cost.
4. Carrying (Holding) cost: The cost associated with carrying (or holding) inventory e.g. Rent for space used for storage, interest on the money locked up, insurance of stored equipment, taxes, depreciation of equipment and furniture used etc.
5. Shortage (Stock out) Cost: The penalty for running out of stock is known as shortage cost. It includes loss of potential profit through sales of items and loss of goodwill, in terms of payment loss of customers.
6. Salvage cost: When demand for certain commodity is affected by the quantity stock, Decision problem is based on profit maximization criteria that includes the revenue from selling. Salvage value may be combined with the cost of shortage and hence it is generally neglected.

3. Factors affecting Inventory Control:

Costs as well as other factors also play an important role in the study of inventory control, which are as follows:

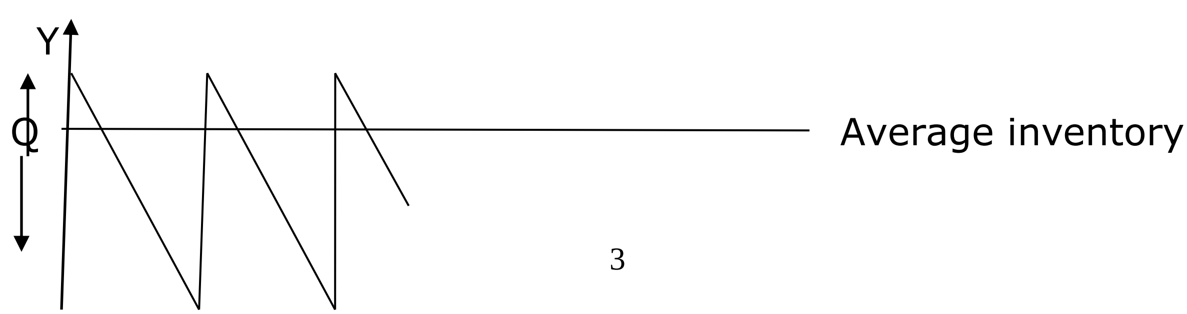
1. Demand: The number of units required per period is called demand. It may be deterministic or probabilistic.
2. Lead Time: Time gap between placing of an order and its actual arrival in the inventory is lead time
3. Order Cycle: Time period between placement of two successive orders is referred to as an order cycle.
4. Time Horizon: Time period over which the inventory level will be controlled is called time horizon.
5. Re-Order level: The level between maximum and minimum stock at which purchasing (or manufacturing) activities must start for replenishment is known as re-order level.

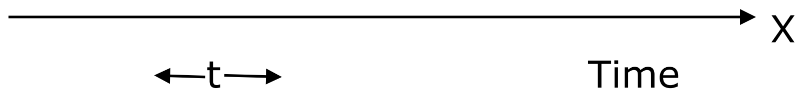
4.Simple Economic Order Quantity Model:

The economic order quantity is that size of order which minimizes total annual costs of carrying inventory and cost of ordering. When the size of order increases, the ordering cost will decrease whereas inventory carrying costs will increase. We make following assumptions in this model.

1. Demand is known and uniform.
2. D=Total number of units purchased/ produced or supplied per time period.
   Q=Lot size in each production.
3. Shortages are not allowed.
4. Production and supply of commodity is instantaneous.
5. Lead time is zero.
6. C_s = set up cost per production run or procurement cost.
7. C_1 = holding cost per unit = I×C, where C = Unit cost, I = Inventory carrying charge expressed as a % of the value of the average inventory

This situation can be shown in the following diagram.





Here it is assumed that after each time t, the quantity Q is produced, purchased or supplied throughout the entire time period. If n denotes the total no. of runs to quantity produced or purchased during the year , then we have

$$1 = n \times t \text{ and } D = n \times Q$$

The average amount of inventory at hand on any day is then Q/2

Therefore, annual inventory holding cost

$f(Q) = (½)Q \times C_1$

And annual cost associated with runs of size Q are given by

$g(Q) = n \times C_s = (D/Q) \times C_s.$

The minimum total cost allows at the point when ordering cost and the total inventory carrying cost are equal.

Therefore, we must have,

$$f(Q) = g(Q)$$

$$(½)QC_1 = (D/Q)C_s$$

$$Q^0 = \sqrt{2DC_s/C_1} \quad \text{Or} \quad Q^0 = \sqrt{2AD/(IC)}$$

is called economic (optimum) lot size.

Hence, we have,

i. Optimum no. of order placed per year

   $= n^0 = D/Q^0$

ii. Optimum length of time between orders

   $t^0 = T/n^0 = T\,Q^0/D = T(2C_s/DC_1)^{1/2}$

   $= (2C_s/DC_1)^{1/2}$ when T is one year

iii. Minimum total annual inventory cost

   $TC^0 = (1/2)Q^0C_1 + DC_s/Q^0$

   $= (2DC_1C_s)^{1/2}$

Example 1

An Engine oil manufacturer purchase lubricants at the rate of Rs. 60 per piece from a vendor. The requirement of these lubricants is 2400 per year. What should be the order quantity per order, if the cost per placement of an order is Rs. 20? Inventory carrying charge per rupee per year is 30 paise.

Solution:

Here, $D = 2400$ per year

C_s = Rs. 20 per order

C = unit cost = 60Rs., I = 0.3 Rs.

Therefore,

$$C_1 = CI = 60 \times 0.3 = \text{Rs. } 18$$

$$Q^0 = \sqrt{2DC_s/C_1}$$

$$= [(2 \times 2400 \times 20)/(18)]^{1/2}$$

$$= \frac{\sqrt{48000}}{3} = 73.0297 \approx 73 \text{ piece of lubricants.}$$

$$\mathbf{t^0 = Q^0/D = 73.0297/2400 = 0.0304 \text{ year} \approx 11 \text{ days}}$$

Minimum average yearly cost = $(2DC_1C_s)^{1/2}$

$= (2 \times 2400 \times 18 \times 20)^{1/2} \approx$ Rs.1314.53 ..(1)

If the company follows policy of ordering every month, then the annual ordering cost is

$= 12C_s$

$= 12 \times 20$ = Rs. 240.

And lot size of inventory each month

$= D/12 = 2400/12 = 200 = Q$

Therefore, average inventory at any time

$= Q/2 = 200/2 = 100$

Storage cost at any time = $100C_1 = 1800$

Total annual cost = Storage cost + Ordering Cost

= 1800 + 240

= 2040 ..(2)

Here minimum average yearly cost {in (1)} is less then the total annual cost {in (2)} so adopt the order at time interval t^0 instead of 200 lubricants each month.

Example 2

A company operating 40 weeks in years about its product of aluminum cable. This cost Rs.150 per meter and there is a demand of 5000meters per week. Each replenishment costs of Rs.600 for administration and Rs. 800 for delivery while holding cost is estimated as 20% of value of held a year. Assuming without shortages, what is the inventory policy of the company?

How would this analysis differ, if the company wanted to maximize profit rather than minimizing the cost? What is the gross profit of company sell cable for Rs. 200 meter?

Solution:

The given information, we convert it into our standard notations.

D=Demand rate=5000 x 40=2,00,000 meter a year

C=purchase cost=Rs. 150 per meter,

Ordering cost (set up cost)=C_s=600+800=Rs.1400

Holding cost=C_1=0.2 x 150=Rs. 30 per meter a year.

Optimal order quantity=$Q^0=(2DC_s/C_1)^{1/2}$

$=[(2 \times 2{,}00{,}000 \times 1400)/30]^{1/2}$

= 4320.4937meter

≈ 4321meter

Total Investment Cost=T_c=Purchase cost+total expected cost.

$= DC + [(1/2)\, Q^0 C_1 + (D/Q_0)\, C_s]$

= 2,00,000 x 150+[(0.5 x 4321 x 30)+(2,00,000/4321) x 1400]

= 3,01,29,614.81

≈Rs.30129615

Here inventory cost are only Rs. 12,965 with a turn over in excess of Rs. 3,00,00,000, which is very small.

If the company wishes to maximize profit the analysis remains same.

The gross profit per unit time becomes

Profit = Revenue – Total Inventory Cost

$= D \times SP - T_c$

= 2,00,000 x 200 – 30129615

= 98,70,385 per year.

5.When Lead-time is not zero:

From the time the order is placed until it is actually supplied is lead time, denoted by L. In most of the business lead time may have some positive value.

We may assume the lead-time is less than the cycle length. But, it is, in general, not true, so define the effective lead time L_e as

$L_e = L - mt^0$, where m is the largest integer not exceeding L/t_0 .

Example 3:

A halozone lamps are replaced at the rate of 50 units per day in a big city. The physical plant orders the lamps periodically. It cost Rs. 120 to initiate a purchase order. A lamp kept in the godown is estimated to cost about Rs. 0.03 per day. The lead- time between placing and receiving an order is 45 days. Determine the optimum inventory policy for ordering halozone lamps.

Solution:

Here,

$D = 50$, $C_s =$ Rs. 120 per order

$C_1 =$Rs. 0.03 per unit per day

$Q^0 = (2DC_s/C_1)^{1/2} = [(2 \times 50 \times 120)/(0.03)]^{1/2}$

$= (4000000)^{1/2}$

$= 2000$ lamps

$t^0 = Q^0/D = 2000/50 = 40$ days

Now, Since the lead time L=45 days, exceeds the cycle length of t^0=40 days.

Compute, $L_e = L - mt^0 = 45 - 1(40) = 5$ days

Here m is the largest integer $\leq L/t^0 = 45/40$ i.e. 1.

Therefore, the re-order point occurs when the inventory level drops to $L_e \times D = 5 \times 50 = 250$ halozone lamps

Hence, the inventory policy for ordering the lamps is order 50 units whenever the inventory level bulbs drops to 250 lamps.

6. When Replenishment (Production) is finite:

In this case, all the assumptions remain same as discussed in section 4, except that the instantaneous replenishment. Assume that the each production run of length t consists of two parts, say t_1 and t_2 such that

i. The inventory is building up at a constant rate of k-r units per unit time during t_1, $k > r$.

ii. There is no replenishment (or production) during time t_2 and the inventory is decreasing at the rate of r per unit time.

i.e. During production period t_1, inventory increasing at the rate of k, and simultaneously decreasing at the rate r. thus, inventory accumulates at the rate of k-r units.

Now t_1 is the time require to producing Q units at the rate k.

$$Q = kt_1 \text{ or } t_1 = Q/k$$

And the maximum inventory level shall be equal to $t_1 (k-r)$.

Therefore average inventory= $(½)\, t_1 (k-r)$

$$= (½)(Q/k)(k-r),\ k > r.$$

If C_1 is the holding cost per unit per year then the total annual holding cost is

$f(Q)$ = Average inventory x C_1

$= (½)\, Q\, (1-r/k) \times C_1$

And annual ordering cost is given by

$$g(Q) = (C_s \times D)/Q$$

Since the minimum holding cost occurs at the point where annual ordering cost and the annual holding cost are equal,

$$f(Q) = g(Q)$$

$$(½)\, Q\, (1-r/k)\, C_1 = (C_s D)/Q$$

$$Q^0 = \sqrt{\frac{2DC_s}{C_1\,(1-\frac{r}{k})}}$$

Hence,

Optimum number of production runs per year $n^0 = D/Q^0$

Optimum length of each lot size production run $t_1^0 = Q^0/k$

Total minimum production inventory cost is given by

$TC^0 = C_sD/Q^0 + (1/2)\, Q^0 \{1-(r/k)\}\, C_1$

$= [2DC_sC_1\{1-(r/k)\}]^{1/2}$

Note:

If $k \to r$ then TC=0 i.e. there will not be holding cost and set up cost.

If $k \to$ infinity it reduces to the problem case(1).

Example 4.

A contractor has to supply 15000 units per day. He can produce 22000 units per day. The cost of holding a unit is Rs. 3 per year and the set up cost per run is Rs150. How frequently and of what size the production runs be made?

Solution:

Here we have given, (assume 300 working days in a year)

$D=15000\times300=45,00,000$ units per year

C_1=Rs. 3 per unit per year.

C_s=Rs. 150 per production run,

r=15,000 units per day

k=22,000 units per day

$Q^0 = [(2DC_s/C_1)(k/(k-r))]^{1/2}$

$= [(2\times45,00,000\times150)/2]^{1/2}\times[(22000)/(22000- 15000)]^{1/2}$

$= 46058.968 \approx 46059$ units

$t^0 = Q^0/r$

$= 3.0706$ day ≈ 3 days

$\boldsymbol{t_0^1}$ **=** length of production run $= Q^0/k$

$= 2.0936 \approx 2$ days

Thus the production run starts at an interval of 3 days and the production continues for 2 days so that in each cycle a batch of 46059 units is produced.

Summary:
Inventory consists of idle resources. The resources may be of any type – materials, machines, goods, men, etc. Inventory control is the process of deciding what and how much of various items are to be kept in stock. We have studied different costs associated with inventory control. We have discussed simple economic ordered quantity model with and without lead time. Also we have seen the inventory model when Replenishments (Production) is finite. Several related applications are presented through examples.

Objective:
To understand the basics of inventory and inventory control. From this talk one can get knowledge regarding inventory models under various setup and conditions. Also able to apply the inventory models in real situations.

Glossary:
Inventory: Inventory means raw materials used in process, finished products, packaging, and offers – stocked in order to meet an expected demand in the future.
Inventory control: It is the process of deciding what and how much of various items are to be kept in stock.
Set up cost: The cost associated with setting up of machinery before starting production.
Ordering cost: The cost of ordering raw material for production purposes. It also includes the cost of advertisements, telephone charges, rent for space used by the purchasing department, travelling expenditure, etc.
Purchase (Production) cost: The cost of purchasing (or producing) a unit of an item is known as purchase cost.
Carrying (Holding) cost: The cost associated with carrying (or holding) inventory

Shortage (Stock out) Cost: The penalty for running out of stock is known as shortage cost.

Lead Time: Time gap between placing of an order and its actual arrival in the inventory is lead time.
Order Cycle: Time period between placement of two successive orders is referred to as an order cycle.
Time Horizon: Time period over which the inventory level will be controlled is called time horizon.

FAQ:

1. What is inventory?

Ans.: Inventory means raw materials used in process, finished products, packaging, and offers – stocked in order to meet an expected demand in the future.

2. What is inventory control?

Ans.: It is the process of deciding what and how much of various items are to be kept in stock.

3. What is holding cost?

Ans.: The cost associated with holding inventory is called holding cost.

4. What is lead time?

Ans.: Time gap between placing of an order and its actual arrival in the inventory is called lead time.

5. What is order cycle?

Ans.: Time period between placement of two successive orders is referred to as an order cycle.

6. What is economic order quantity ?

Ans.: The economic order quantity is that size of order which minimizes total annual costs of carrying inventory and cost of ordering.

7. When the total cost of inventory becomes minimum?

Ans.: The minimum total cost occurs at the point when ordering cost and the total inventory carrying cost are equal.

8. For standard economic inventory model state the formula of economic lot size in usual notations.

Ans.: Economic lot size = $Q^0 = \sqrt{2DC_s/C_1}$.

9. When Replenishment(Production) is finite state the formula of economic lot size in usual notations.

Ans.: Economic lot size = $Q^0 = \sqrt{\frac{2DC_s}{C_1\,(1-\frac{r}{k})}}$

M.C.Q.

1. What is the full form of EOQ?
   (a) Economic order quality
   (b) Economic order quantity
   (c) Economic order queue
   (d) All of the above

2. Which one is true
   (a) Cost associated with the ordering of units is called set up cost
   (b) Cost of ordering units is called production cost
   (c) Cost of carrying the units is called holding cost
   (d) Penalty cost for running out of stock is salvage cost

3. Lead time means
   (a) Time between the start and end of the production units

(b) Time between starting of production and the time of stock becoming zero.

(c) Time between placing an order and its arrival in the inventory.

(d) Time starting of first trial production.

4. Minimum total cost occurs at the point where

(a) Ordering cost and the total inventory carrying cost are equal.

(b) Ordering cost is greater than the total inventory carrying cost.

(c) Ordering cost is smaller than the total inventory carrying cost.

(d) Ordering cost is half of the total inventory carrying cost

5. For the fundamental EOQ problem, the economic order value is given by

(a) $(2DC_1/C_s)^{1/2}$

(b) $(2DC_s/C_1)^{1/2}$

(c) $(C_1C_s/2D)^{1/2}$

(d) $(DC_1/2C_s)^{1/2}$

6. For the fundamental EOQ problem, the optimum length of the time between order is given by

(a) T/n^0

(b) DC_s/n^0C_1

(c) D/n^0

(d) Q/n^0

7. Which of the following is not correct, In the content of the inventory management the fellow need to be addressed

(a) How much to order?

(b) When to order?

(c) How much safety stock to keep?
(d) Maximum of the total inventory cost.

7. Which of the following is not an assumption underlying the fundamental problem of EOQ
(a) Demand is known and uniform.
(b) Holding cost per unit per time is constant.
(c) Shortages are not allowed.
(d) Lead time is not zero.

9. Which of the following is not correct
(a) Procurement cost is low if items are produced frequently in small lots
(b) Procurement cost is less if items are produced less frequently and in big lots
(c) In the fundamental problem of EOQ, Total inventory cost is the sum of the ordering and the holding cost.
(d) Inventory carrying cost falls when the Quantity ordered per order is small.

Assignments

1. What is inventory?
2. What is inventory control?
3. Define lead time
4. Define set up cost
5. Define holding cost
6. Define shortage cost
7. Define penalty cost
8. Define order cycle
9. Define reorder level
10. State the assumptions for fundamental EOQ problems
11. State the condition for total cost to be minimum inventory.

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Links:

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