Simulation

[Academic Script]

Subject	**:**	**Business Economics**
Course	**:**	**B.A., 4th Semester, Undergraduate**
Paper No.	**:**	**403**
& Title	**:**	**Quantitative Techniques for Management**
Topic No.	**:**	**A3**
& Title	**:**	**CPM/PERT Analysis, Simulation, Simple Inventory Models**
Title	**:**	**Simulation**

Credits

Subject Co-ordinator

Dr. M. N. Patel
Professor, Dept. of Statistics,
I/c Director,
K. S. School of Business Management,
Gujarat University,
Ahmedabad.

Subject Expert

Dr. M. N. Patel
Professor, Dept. of Statistics,
I/c Director,
K. S. School of Business Management,
Gujarat University,
Ahmedabad.

Technical Asst. & Sound Recording

Archana Patel
Smita Bhatt

Multimedia

Gaurang Sondarva

Camera

Maqbool Chavda

Technician

Kirit Dave

Floor Assistant

Hemant Upadhyaya

Helper

Bharat Chauhan

Jagdish Jadeja

Editing & Graphics

Akash Choudhary

Research

Prof. Pooja Khatri

K. S. School of Business Management,

Gujarat University,

Ahmedabad.

Producer

Dr. P. P. Prajapati

Dinesh Goswami

1. Introduction to Simulation:

In this talk we will study about simulation.

Simulation is a numerical technique for conducting experiments that involve certain types of mathematical and logical relationship used to understand the behavior for the operation of the system. When it is difficult to observe the actual real data simulation is the only available method to generate such data.

2. Methodology of Simulation:

i. Identify and define the problem clearly.

ii. List the statement of objectives of the problem

iii. Formulate the variables that influence the situation.

iv. Obtain a consistent set of values(or states) for the variables.

v. Use the sample obtained in step ii to calculate the value of the
decision criterion by using the relationships among the
variables for each of the alternative decisions.

vi. Repeat the steps ii and iii until a sufficient number of samples
are available.
vii. Tabulate the various values of the decision criterion and
choose the best policy.

3. Need for simulation:

Analyst can define the constants and variables related to the problem under study by considering simulation. He can setup the possible courses of action and establish the policy or rule as a measure of effectiveness. The following are the reasons for conducting simulation for the OR problems.

i. When for the original experiment replication is not permissible, experiments are very costly or destructive.
ii. When a mathematical model involved in the problem is very complex to solve it.
iii. To collect detail information about all the decision variables involved in the problem is not possible by experimenting on the real system.

iv. When it is difficult to observe the actual reality.
v. When it is impossible to develop a mathematical model without appropriate assumption.
vi. Sufficient time is not available to operate an experiment very long period of a time.

4. Types of simulation:

There are four types of simulation models.

i. deterministic ii. probabilistic iii. static iv. dynamic

i. In deterministic models, the variables are not random variables
and the models are described by exact functional relationships.

ii. In probabilistic models, the method of random sampling is
used. The method used for such models is known as " Monte -
Carlo Technique".

iii. In static models, the models do not take variable time into consideration.

iv. Dynamic models deal with time-varying interaction.

Example 1. (Deterministic model)

Customers arrive ata bank counter for the required service. Assume that inter-arrival times are constant with 1.5 time units and service times are also constant with 3 time units. Simulate the system for 10.5 time units.

What is the average waiting time per customer?

What is the percentage idle time of the facility?

Assume that the system starts at time t=0.

Solution:

In the beginning, since the service facility is free. i.e. for first customer no waiting time, service starts at time t=0 and its departure time is t = 0 + 3 = 3.

Next arrival occur at time t = 0 + 1.5, which is stored before E_d (event of departure) at t = 3.

Now, since the facility is still busy, customer 2 is put in the queue and is first to be considered in this queue. A new arrival customer 3 occurred at t = 1.5+1.5 = 3, at this time unit, first customer departs which leaves the facility free, customer 2, who was the first to join the queue, now gets service. The waiting time is computedas the time period from the instant he joined the queue until he commences service. The procedure is repeated until the simulation period is completed. The results of simulation are given in the table.

Time Event Customer arriveWaiting time

0.0 E_a1 -

1.5 E_a2

3.0 E_a& E_{d1} 3 for “2” 3.0 – 1.5 =1.5

4.5 E_a& E_{d2}4 for “3” 4.5 – 3.0 =1.5

6.0E_a& E_{d3} 5 for “4” 6.0 – 4.5

=1.5

7.5 E_a& E_{d4}6 for “5” 7.5 – 6.0 =1.5

9.0 E_{d5}7 – th customer for “6” 9.0 – 7.5 =

1.5 is not allowed

is not allowed

10.5End --- --

-

Hence the average waiting time per customer is

$\frac{0+1.5+1.5+1.5+1.5+1.5}{6} = \frac{7.5}{6} = 1.25$

Percentage idle time of the facility = 0%

Example 2. (Probabilistic model)

Daily Demand	**Probability**	**Cumm. Probability**	**Random Number Range**	**Random Numbers**	**Demand**
0	**0.01**	**0.01**	**00-00**	**35**	**20**
10	**0.20**	**0.21**	**01-20**	**92**	**40**
20	**0.15**	**0.36**	**21-35**	**68**	**30**
30	**0.40**	**0.76**	**36-75**	**3**	**10**
40	**0.20**	**0.96**	**76-95**	**51**	**30**
50	**0.04**	**1.00**	**96-99**	**5**	**10**

	72	**30**
	84	**40**
	98	**50**
	34	**20**

A Bakery keep stock of a popular brand of a cake. Previous experience indicates the daily demand as given below:

Daily demand: 0 10 20 30 40 50

Probability: 0.01 0.20 0.15 0.40 0.20 0.04

Generate 10 days demand. Find out the stock situation if the owner of the bakery decides to make 30 cakes every day. Estimate the daily average demand for the cakes on the basis of the simulated data.

Solution:

We prepare the following table.

Now we write 10 two digits random numbers from the random number tables as below:

35, 92, 68, 03, 51, 05, 72, 84, 98, 34

as random number 35 belongs to the range 21 – 35, corresponding demand is 20; the random number 92 belongs to the range 76 – 95 and corresponding demand is 40 and so on we generate the demand for each given random numbers. Thus demand for ten days and remaining stock (cakes on hand – demand)will be given below.

Day	Random Number	Demand	Stock at the end of the day
1	**35**	**20**	**30-20=10**
2	**92**	**40**	**10+30-40=0**
3	**68**	**30**	**0+30-30=0**
4	**3**	**10**	**0+30-10=20**
5	**51**	**30**	**20+30-30=20**
6	**5**	**10**	**20+30-10=40**
7	**72**	**30**	**40+30-30=40**
8	**84**	**40**	**40+30-40=30**
9	**98**	**50**	**30+30-50=10**
10	**34**	**20**	**10+30-20=20**

$$\text{Averagedailydemand} = \frac{20+40+30+10+30+10+30+40+50+20}{10}$$

$$= \frac{280}{10} = 28\ cakes$$

5. Monte - Carlo Simulation:

The Monte – Carlo method is a simulation technique in which statistical distribution functions are created by using a series of random numbers. This approach has the ability to develop a very large amount of data within a very short time by a computer programming. Such a method is helpful when mathematical model is not possible for the problem and it is not possible to solve the problem by any analytical method.

A solution obtained by Monte-Carlo simulation is very close to the optimal value depending on the number of simulated trials which tends to infinity.

Following are the steps for Monte- Carlo simulation:

i. Define the problem.
ii. Specifying the variables and parameters formulate the appropriate decision rules.

iii. Identify the theoretical or empirical distribution to state the patterns the occurrence associated with the variables.
iv. Define the relationship between the variables and parameters
v. State if any condition to start simulation and decide the number of runs of simulation to carry out the necessary calculation.
vi. Select a random number generator and create a random number to use for simulation.
vii. Use the generated random numbers to the factors identified in defining the problem.
viii. Summarize the results.
ix. Evaluate the results.
x. Hence prepare a list of course of action and advise to management.

Example 3. In a city rain on today depends on the rained on the previous day or not. The probability distribution of rain are given below:

(i) If there was rain on the previous day.
Event: no rain 1 cm 2 cm 3cm 4cm 5cm

Probability: 0.40 0.30 0.15 0.10
0.040.01

(ii) If there was no rain on the previous day.

Event: no rain 1 cm 2 cm 3cm
4cm

Probability: 0.60 0.20 0.15 0.03
0.02

Simulate the city's weather for 10 days and determine the average rainfall of the 1o days, percentage of days without rain using the following random numbers.

76, 36, 65, 52, 02, 68, 72, 50,
04, 82

Assume that there was no rain on the day before the first day of simulation.

Solution:

(i) Previous day rain distribution :

Event Prob. Cum. prob. Random number
range

No rain 0.40 0.40 00
– 39

1 cm 0.30 0.70 40
– 69

2 cm	0.15	0.85	70 – 84
3 cm	0.10	0.95	85 – 94
4 cm	0.04	0.99	95 – 98
5 cm	0.01	1.00	99 - 99

(ii) Previous day no rain distribution:

Event	Prob.	Cum. prob.	Random number range
No rain	0.60	0.60	00 – 59
1 cm	0.20	0.80	60 – 79
2 cm	0.15	0.95	80 – 94
3 cm	0.03	0.98	95 – 97
4 cm	0.02	1.00	98 – 99

(iii) Simulation of rain for 10 days:

Day 1Randomnumber 76

Event: By the assumption of no rain on previous day we
see in Table (ii) where random number 76 belongs to the range 60 – 79,which shows a rain '1 cm'.

Rain Event: 1 cm

Day 2 Random number 36

Event: Now there is a rain on the previous day so we see
in the table (i) where random number 36belongs to the range 00 – 39 , which shows 'no rain'.

Rain Event: No rain

Day 3 Random number 65

Event:See in table (ii), random number 65 belongs to
therange 60 – 79, whichshows rain '1 cm '.

Rain Event: 1 cm

Day 4 Random number52Rain Event: 1 cm

Day 5 Random number02Rain Event: No rain

Day 6 Random number 68 Rain Event: 1 cm

Day 7 Random number 72Rain Event: 2 cm

Day 8 Random number50Rain Event: 1 cm

Day 9 Random number04Rain Event: No rain

Day 10 Random number82 Rain Event: 2 cm

Total rain = 9 cm

Average rain = 9/10 = 0.9 cm

Percentage of days without rain = (3/10)x100 = 30%

6. Random sample from distribution:

Now we consider some examples to generate random samples from the given distributions.

Example 4.

Let X= number of accidents on a day at a particular place of a city follows Poisson distribution with average number of accidents 2 per day. Generate number of accidents for 1o days using the random numbers given below.

287, 952, 045, 101, 520, 764, 828, 479, 122, 335

Solution:

Here X is a random variable for the number of accidents per day, which has Poisson distribution given by

$$P(x) = \frac{e^{-m} m^{x}}{x!}\ , x = 0,1,2, \ldots .., m\ = 2$$

x P(x) Cum.Prob.Rand.No. Rand.No. Simul.no.of

Range accidents

0 0.1350.135 000 – 134 287 1

1 0.271 0.406 135 – 405 952

5

2 0.271 0.677 406 – 676 045

0

3 0.018 0.857 677 – 856 101

0

4 0.090 0.947 857 – 946 520

2

5 0.036 0.983 947 – 982 764

3

6 0.012 0.995 983 – 994 828

3

≥70.005 1.000 995 – 999 479 2

122 0

335 1

Example 5.

The life of an electric bulb follows exponential distribution with mean life1500 hours. Simulate life times of such 10 electric bulbs.

Solution:

Let X = life time of electric bulb,its probability density function is given by

$$f(x) = \frac{1}{\theta} e^{-x/\theta}, x > 0, \theta > 0.$$

Here θ = mean life time = 1500

To generate lifetime we must have cumulative distribution function.

We obtain cumulative distribution function as follow.

F (x) = P (X ≤ x) = $\int_0^x \frac{1}{\theta} e^{-y/\theta} dy$

$$= \frac{1}{\theta} \int_0^x e^{-y/\theta} dy$$

$$= \frac{1}{\theta} \left[\frac{e^{-y/\theta}}{-\frac{1}{\theta}} \right]_0^x = 1 - e^{-x/\theta}$$

Thus, 1- F(x) = $e^{-x/\theta}$ ⇒Ln[1-F(x)] = -x/θ

⇒x = -θLn[1-F(x)] .. (1)

Replacing F(x) by random number in (1), we get lifetime of an electric bulb x.

The following table gives random numbers and corresponding simulated life times.

Random number	**Life Time(x) = -1500Ln[1-Rand.]**
0.135	**217.5387**

0.008	**12.04826**
0.427	**835.3043**
0.198	**330.9700**
0.625	**1471.244**
0.739	**2014.852**
0.525	**1116.661**
0.263	**457.7511**
0.348	**641.5661**

Example 6.

Let X = number of defective blades in a box of 8 blades. The probability of a defective blade in a produced lot is 0.2.

Generate number of defective blades in such 10 boxes using the following random numbers.

0347, 9774, 1676, 1256, 5559, 1622, 8442, 6301, 3321, 5760

Solution:

Here X follows binomial distribution with parameters n = 8 and p = 0.2. Its P(x) is given by

$$p(x) = \binom{8}{x}(0.2)^x(0.8)^{10-x}, x = 0, 1, \dots, 8$$

We use the method discussed in example 4.

x	p(x)	Cum. p(x)	Random No. range	Random No.	Simulated No. of defective blades
0	0.0576	0.0576	0000 -0575	0347	0
1	0.1977	0.2553	0576 - 2552	9774	5
2	0.2965	0.5518	2553 - 5517	1676	1
3	0.2541	0.8059	5518- 8058	1256	1
4	0.1361	0.9420	8059 - 9419	5559	3
5	0.0467	0.9887	9420 - 9886	1622	1
6	0.0100	0.998	9887 -	8842	4

			86		
7	0.0012	0.999	9987 - 98	6301	3
8	0.0001	1.000	9999 - 99	3321	2
				5760	3

Summary:

Simulation is a numerical technique for conducting experiments that involve certain types of mathematical and logical relationship used to understand the behavior for the operation of the system. When it is difficult to observe the actual real data simulation is the only available method to generate such data. There are four types of simulation models.

i. deterministic ii. probabilistic iii. static iv. dynamic

The Monte – Carlo method is a simulation technique in which statistical distribution functions are created by using a series of random numbers. Such a method is helpful when mathematical model is not possible for the problem and it is not possible to solve the problem by any analytical method.
A solution obtained by Monte-Carlo simulation is very close to the optimal value depending on the number of simulated trials which tends to infinity.