[Academic Script]

Quantitative Techniques for Management

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PERT - CPM

Programme evaluation review technique (PERT) and Critical path method (CPM) are two basic planning and controlling techniques that utilize a network to complete a pre-determined project or schedule. These are the methods of minimizing the trouble spots such as production delays and interruptions, by determining critical factors and coordinating various parts of the overall job.

To understand the network techniques let us be familiar with a few basic terms of which both PERT and CPM are special applications:

1) Activity

   An activity represents some action and is a time consuming effort necessary to complete a particular path of the overall project thus each and every activity has a point of time where it begins and a point where it ends.

2) Event

   The beginning and end point of an activity are called event or nodes .Event is a point in time and does not consume any resources. It is represented by numbered circle.

3) Network

   It is a graphical representation of logically and sequentially connected arrows and nodes, representing activities and event in a project. It is also called arrow diagram.

4) Dummy activity

   Certain activities which neither consume time nor resources but are used to simply represent a connection or a link between the events are known as dummies. It is shown in the network by dotted lines. The purpose of introducing dummy activity is to maintain uniqueness in the numbering system and to maintain a proper logic in the network.

5) Critical Path
Critical Path is the longest path through the project network: the activities on the path are the critical activities therefore any delay in their completion must be avoided to prevent delay in project completion

Network construction

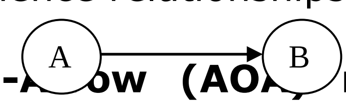
There are number of rules in connection with the handling of events and activities of a project network that should be followed:

1) Try to avoid the arrows that cross each other
2) Use straight arrows
3) No event can occur until every activity preceding it has been completed
4) An event cannot occur twice
5) An activity succeeding an event cannot be started until that event has occurred.
6) Use arrows from left to right
7) Dummies should be introduced only if it is extremely necessary.
8) The network has only one entry point called the start event and one point of emergence called the end event.

Network model use the following two types of precedence network to show precedence requirements of the activities in the project

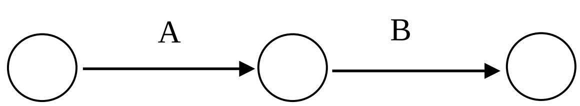
1) Activity-on-Node (AON) network: In this Network, each activity is represented by a node i.e. circle or rectangle. The arrows show the precedence relationships between the activities.

B must follow A



2) Activity-on-Arrow (AOA) network: In the Network, each activity is represented by an arrow.

B must follow A



Fulkerson's Rule (Numbering the Events)

After the network is drawn in a logical sequence, every event is assigned a unique number. The number sequence must be such as to reflect the flow of the network. In numbering the events, the following rules should be observed

- Event numbers should be unique.
- Event numbering should be carried out on a sequential basis, from left to right.
- The initial event, which has all outgoing arrows with no incoming arrow, is numbered as 1.
- Delete all the arrows emerging from all the numbered events. This will create at least one new start event, out of the preceding events.
- Number all new start events 2, 3 and so on. Repeat this process until the terminal event without any successor activity is reached. Number the terminal node suitability.

Time Estimates in network Analysis

- Once the network of a project is constructed, the time analysis of the network becomes essential for planning various activities of the project. Activity time is a forecast of the time for an activity which is expected to take from its starting point to its completion.
- Consider the following notations for the purpose of calculating various time events and activities.

- E_i=Earliest Occurrence time of an event i.
- L_i=Latest allowable time of an event i.
- ES_{ij}=Early starting time of an activity (i, j).
- LS_{ij}=Late starting time of an activity (i, j).
- EF_{ij}=Early finishing time of an activity (i, j)
- LF_{ij}=Late finishing time of an activity (i, j)
- t_{ij}=Duration of an activity (i, j).

- **Float of an Activity and Event**
- The float of free time is the length of time in which a non-critical activity and/or an event can be delayed or extended without delaying the total project completion time.
- Mainly three types of floats are defined for each non-critical activity of the project.
- 1) Total Float: This is the length of time by which an activity can be delayed when all preceding activities are completed at their earliest possible time and all successor activities can be delayed until their latest permissible time.
- **Total Float (TF_{ij}) = LF_{ij}- EF_{ij}**

- 2) Free float: The free float of a non-critical activity is defined as the time by which the completion of an activity can be delayed without causing any delay in its immediate succeeding activities. Free float values for each activity (i, j) are computed as
- **Free float (FF_{ij}) = (E_j-E_i)- t_{ij}**

= Min {ES_{ij}, for all immediate successors of activity (i, j)}- EF_{ij}

- 3) Independent Float: This is the amount of acceptable delay in the completion of an activity so that it neither affects its predecessor nor the successor activities.
- Independent float values for each activity (i, j) are computed as

Independent float (IF_{ij})= {ES_{ij}-LS_{ij}}-t_{ij}

- The relation between this three float can be given as

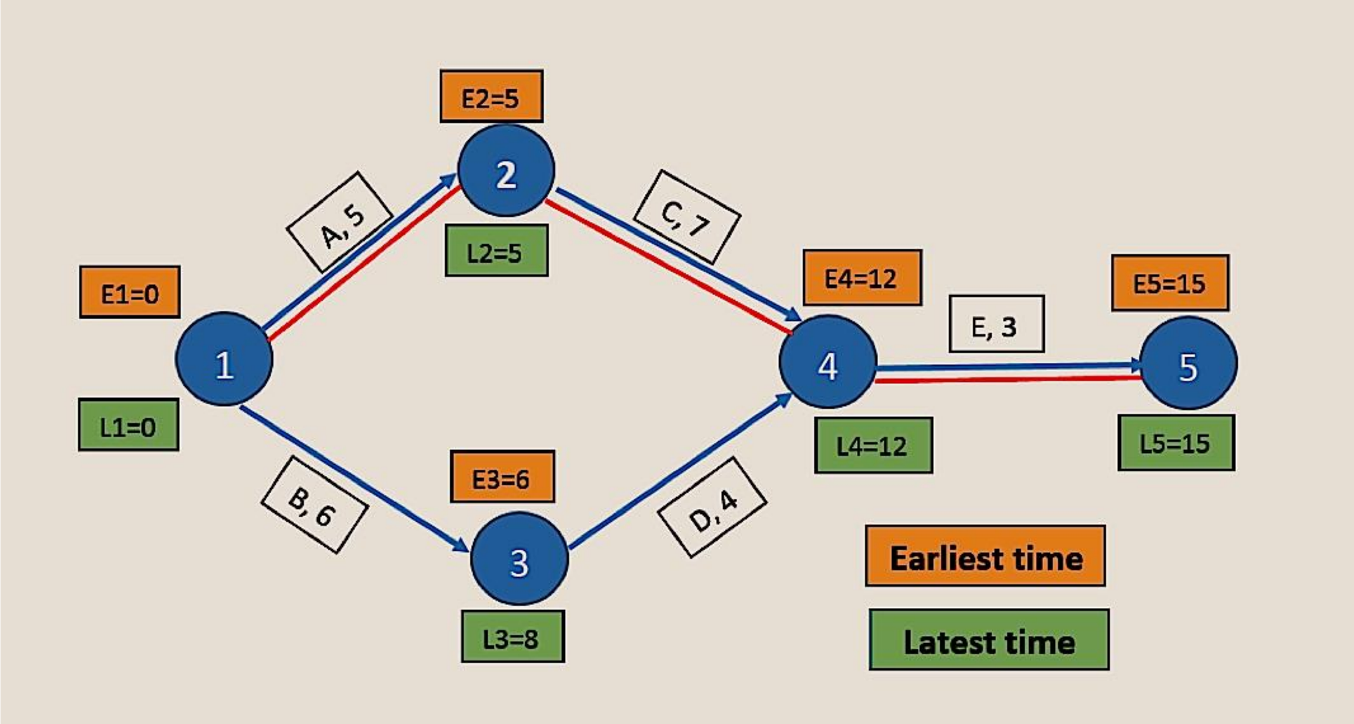
- Independent float $\leq$ Free float $\leq$ Total float

Example:
Construct a network for the project whose activities and precedence relationships are as given below and determine critical path:

Activities	A	B	C	D	E

Immediate Predecessor	-	-	A	B	C, D
Time	5	6	7	4	3

Solution:



The earliest occurrence time (E) and the latest occurrence time (L) of each event is computed as

E1=0

E2=E1+duration of activity 1-2=0+5=5

E3=E1+duration of activity 1-3=0+6=6

E4 =MAX [E2+duration of activity 2-7, E3+duration of activity 3-4]
=MAX [5+7, 6+4]=Max [12, 10]=12

E5=E4+duration of activity 4-5=12+3=15

To determine the latest expected time, we start with E_5, being the last event and move backwards-subtracting t_e from each activity.

L5=15

L4=L5-duration of activity 4-5=15-3=12

L3=L4-duration of activity 3-4=12-4=8

L2= L4-duration of activity 2-4 =12-7=5

L1=MIN [L2-duration of activity 1-2, L3-duration of activity 1-3]
=MIN [5-5, 8-6]

=MIN [0, 2]

=0

The network analysis table is compiled as

		Start time		Finish Time		
Activity	**Duration**	**Earliest**	**Latest**	**Earliest**	**Latest**	**Total float**
1-2	**5**	**0**	**0**	**5**	**5**	**0**
1-3	**6**	**0**	**0**	**6**	**8**	**2**
2-4	**7**	**5**	**5**	**12**	**12**	**0**
3-4	**4**	**6**	**8**	**10**	**12**	**2**
4-5	**3**	**12**	**12**	**15**	**15**	**0**

Total Float (TF_{ij}) = LF_{ij} - EF_{ij}

EF_{ij}=Early finishing time of an activity (i, j)

LF_{ij}=Latest finishing time of an activity (i, j)

So 5-5=0 and same way other float is calculated.

Activity 1-2, 2-4, 4-5 having zero float are the critical activities. So the critical path is 1-2-4-5.

Programme Evaluation and Review Technique (PERT)

PERT is used when the activities are non-deterministic in nature. It is a probabilistic method, where the activity times are represented by a probability distribution. This distribution of activity times is based on three different time estimates made for each activity, which are

1. **Optimistic time estimate**
2. **Most likely time estimate**
3. **Pessimistic time estimate**

1. Optimistic Time estimate: It is the smallest time taken to complete the activity, if everything goes well. There is very little chance that an activity can be completed in a time less than the optimistic time. It is denoted by t_o or a.

2. Most likely time Estimate: It refers to the estimate of the normal time the activity would take. This assumes normal delays. It is the mode of the probability distribution. It is denoted by t_m or m.

3. Pessimistic time Estimate: It is the longest time that an activity would take, if everything goes wrong. It is denoted by **t_P or b.**

   For these three time estimates, we have to calculate the expected time of an activity. It is given by the weighted average of the three time estimates

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Variance of the activity is given by,

$$\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$$

The expected length (duration), denoted by T_c of the entire project is the length of the critical path, i.e. the sum of the $t_c's$ of all the activities along the critical path.

The main objective of the analysis through PERT is to find the completion date for a particular event within the specified date T_s, given by $P\,(Z \le D)$ where,

$$D = \frac{Due\ date - Expected\ date\ of\ completion}{\sqrt{Project\ variance}}$$

Here, Z stands for standard normal variable.

PERT Procedure

Step-1: Draw the project Network.

Step-2: Compute the expected duration of each activity using the formula,

$$t_e = \frac{t_o + 4t_m + t_p}{6}$$

Also calculate the expected variance $\sigma^2 = \left[\frac{t_p - t_o}{6}\right]^2$

of each activity.

Step-3: Compute the earliest start, earliest finish, latest start, latest finish time and total float of each activity.

Step-4: Find the critical path and identify the critical activities.

Step-5: Compute the project length variance σ^2, which is the sum of the variance of all the critical activities and hence, find the standard deviation of the project length σ.

Step-6: Calculate the standard normal variable $Z = \frac{T_s - T_e}{\sigma}$,

where T_s is the scheduled time to complete the project. T_e=Normal expected project length duration.

σ= Expected standard deviation of the project length

Using the normal curve, we can estimate the probability of completing the project within a specified time.

Example:

A small project is composed of seven activities, whose time estimates are listed in table as follows:

Activity	**Estimated Duration (weeks)**		
	Optimistic (a)	**Most likely (m)**	**Pessimistic (b)**
1-2	**1**	**1**	**7**
1-3	**1**	**4**	**7**
1-4	**2**	**2**	**8**
2-5	**1**	**1**	**1**
3-5	**2**	**5**	**14**

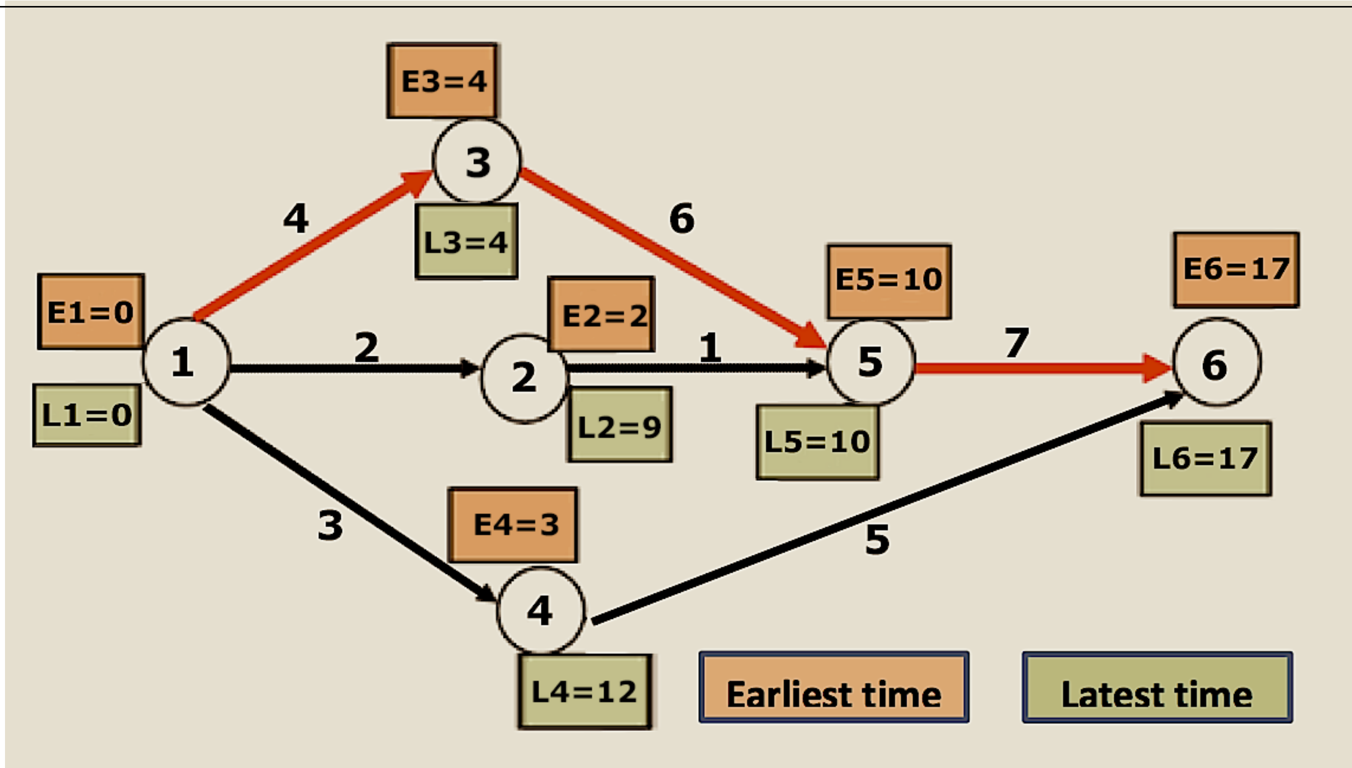
4-6	**2**	**5**	**8**
5-6	**3**	**6**	**15**

You are required to:

1. Draw the project network.
2. Find the expected duration and variance of each activity.
3. Calculate the earliest and latest occurrence for each event and the expected project length.
4. Calculate the variance and standard deviation of project length.
5. What is the probability that the project will be completed 4 weeks earlier than expected?

Solution:
The expected time and variance of each activity is computed as

Activity	**Optimistic (a)**	**Most likely (m)**	**Pessimistic (b)**	$t_e = \frac{a+4m+b}{6}$	$\sigma^2 = \left[\frac{b-a}{6}\right]^2$
1-2	**1**	**1**	**7**	**2**	**1**
1-3	**1**	**4**	**7**	**4**	**1**
1-4	**2**	**2**	**8**	**3**	**1**
2-5	**1**	**1**	**1**	**1**	**0**
3-5	**2**	**5**	**14**	**6**	**4**
4-6	**2**	**5**	**8**	**5**	**1**
5-6	**3**	**6**	**15**	**7**	**4**



The earliest and the latest occurrence time for each is calculated as

$E_1 = 0$

$E_2 = 0 + 2 = 2$

$E_3 = 0 + 4 = 4$

$E_4 = 0 + 3 = 3$

$E_5 = Max(2 + 1, 4 + 6) = 10$

$E_6 = Max(10 + 7, 3 + 5) = 17$

To determine the latest expected time, we start with E_6 being the last event and move backwards-subtracting t_e from each activity.

Hence, we have,

$L_6 = E_6 = 17$ $\quad$ $L_5 = L_6 - 7 = 17 - 7 = 10$

$L_4 = 17 - 5 = 12$ $\quad$ $L_3 = 10 - 6 = 4$

$L_2 = 10 - 1 = 9$

$L_1 = Min(9 - 2, 4 - 4, 12 - 3) = 0$

We observe the critical path of the above network as 1-3-5-6.

The expected project duration is

1-3=4,

3-5=6,

5-6=7

So $T_e = 4 + 6 + 7 = 17\ weeks$

The variance of the project length is given by**,**

1-3=1,

3-5=4,

5-6=4 $\sigma^2 = 1 + 4 + 4 = 9$

The standard deviation of the project is given by 3.

The probability of completing the project within 4 weeks earlier than expected is given by

$$P(Z \leq D), where\ D = \frac{T_s - T_e}{\sigma}$$

$$D = \frac{Due\ date - Expected\ date\ of\ completion}{\sqrt{Project\ variance}}$$

$$D = \frac{(17-4)-17}{3} = \frac{13-17}{3} = \frac{-4}{3} = -1.33$$

Therefore, $P(Z \leq -1.33) = 0.5 - P(0 \leq\ Z \leq\ 1.33)$

$$= 0.5 - 0.4082\ (from\ st.normal\ dist.table)$$

$$= 0.0918 = 9.18\%$$

Conclusion:

If the project is performed 100 times under the same condition, then there will be 9 occasions for this job to be completed in 4 weeks earlier than expected.

Project Cost

In order to include the cost factors in project scheduling, first define the cost duration relationships for various activities in the project. The total cost of any project comprises of direct and indirect cost.

Cost Slope

The cost slope, indicating the increase in cost per unit reduction in time is defined as,

Cost slope$=\frac{Crash\ cost - normal\ cost}{Normal\ time - crash\ time} = \frac{C_c - C_N}{T_N - T_C}$

i.e. it represents the rate of increase in the cost of performing the activity per unit reduction in time and is called cost/time trade off. It varies from activity to activity. The total project cost is the sum total of the project's direct and indirect costs.

Time-Cost Trade-off Procedure

The method of establishing time-cost trade-off for the completion of a project can be summarized as:

Step-1: Determine the normal project completion time and associated critical path for the two cases

1. When all critical activities are completed with their normal time. This provides the starting point for crashing analysis.
2. When all critical activities are crashed. This provides the stopping point for crashing analysis.

Step-2: Identify critical activities and compute the cost slope for each of these.

The values of cost slope for critical activities indicate the direct extra cost required to execute an activity per unit of time.

Step-3: For reducing the total project completion time, identify and crash an activity time on the critical path with lowest cost slope value to the point where

a. Another path in the network becomes critical, or
b. The activity has been crash to its lowest possible time.

Step-4: if the critical path under crashing is still critical, return to step 3. However, if due to crashing of an activity time in step-3, other path(s) in the network also become critical, then identify and crash the activity(s) on the critical path(s) with the minimum joint cost slope.

Step-5: terminate the procedure when each critical activity has been crashed to its lower possible time. Determine total project cost corresponding to different project duration.

By using this procedure we can solve the examples of crashing.