[Academic Script]

Introduction to Operations Research

Subject:	Business Economics
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Academic Script

1. Introduction and Meaning

The term OR, was first used by MC Closky and Trefthen in 1940. In military context new scientific methodology was used during World War II. For solving strategic and technical problem such methods were used. Such new approach used in the scientific study of the operations of the system was known as operations research (we will abbreviated as OR).

OR was used as a one of the subjects in the universities during the year 1950. Then after it becomes very much useful in economics, management, public administration, Engineering and in many more branches. In 1950 OR society of America came in to existence. International federation of OR societies were established in 1957. In India at Regional Research laboratory at Hyderabad and unit of OR was opened in 1949. During the same time defense science laboratory was started another group to handle the problems regarding stores, purchase and planning. Indian Statistical Institute, Calcutta used the application OR methods in national planning and survey in 1957. In India prof. Mahalanobis formulated the second five year plan with the help of OR techniques. The linear programming techniques of OR become most popular in India.

OR has been described by various persons as follows:

(i) OR is a scientific method of providing executives departments with a quantitative basis for decision under their control ---- P.M.Morse and G.E.Kimball.

(ii) OR is a scientific approach to problems solving for executive management ---H.M.Wagner.

(iii) OR is a scientific knowledge through interdisciplinary team effort for the purpose of determining the best utilizations of limited recourses -----H.A.Taha.

2. Opportunities of OR

(i) The decision-maker can get a solution to his problem with the help of models of OR.

(ii) A manager can determine the best decision for the organization as whole by considering all necessary variables associated with the decision using OR approach.

(iii) OR approach becomes useful to a decision maker to solve a complex problem which may not be possible to solve by any traditional methods.

(iv) Using OR approach a simulation a decision maker can examine a situation from different angles and different conditions associated to the real problem.

3. Applications of OR

OR techniques are used in many different areas of research in defense, Government, Industry etc.

Some of the applications are describe as below:

(i) Finance, Budgeting and Investment:

In cash flow analysis, in dividend policies, in investment portfolios

(ii) Purchasing, Procurement and exploration:

- In deciding rules for buying and supplies in stable or varying prices.
- To determine quantities and timing to purchase.
- Replacement policies.

(iii) Marketing

- For product selection, timing and competitive actions.
- In advertising media with respect to cost and time for.

(iv) Production management

- To determine location and size of warehouses, distribution centers and retail outlets.
- To determine the transport schedule.
- In production scheduling and sequencing.
- In maintenance policies and preventive maintenance

(v) Research and Development

- To determine the area of research and development
- To determine time cost trade off
- To coordinate multiple research projects.

(vi) LIC: In the cases of policies with profits, To determine best way of distributing profit.

4. Limitations of OR

(i) There are certain problems can be solved least cost and less sophisticated approaches available and decision makes may have to solve such problems only once. In such situation use by complex OR model becomes too expensive.

(ii) Sometimes OR users become too much enamored with the model they have built and forget the fact that the model does not represent the real world problem.

(iii) Sometimes the basic data are subject to frequent changes. In such cases modification of OR models becomes very much costly.

(iv) Many OR models cannot be solved without using the computer and becomes difficult to explain the outcomes of the models.

(v) Magnitude of computation and luck of consideration for non-quantifiable factors.

5. Modeling in OR

A model in OR is a simplified representation of an actual object or situation. It shows the relationships and inter-relationships of action and reaction in terms of cause and effects.

A. Object of Model: The objective of modeling is to provide a means for analyzing the behavior of system to improve its performance.

B. Main characteristics of a good model:

(i) It should be capable of taking in to account new formulations without affecting in its trine.

(ii) Assumption involved in the model should be small possible.

(iii) It should be simple and coherent.

(iv) Numbers of variables involved in the model should be less.

(v) It should be open to parametric type of treatment.

(vi) It should not take much time in its construction for a given problems.

C. Limitations of a model:

(i) Model should just not be considered as absolute in any sense, it is just an attempt to understand the operation.

(ii) Validity of any model can only be verified by carrying the experiment and relevant data characteristics.

D. Classification of models:

Several types of models are used in area of business or industrial activity.

(i) Models by degree of abstraction: These models are based on the past data of the problems under consideration.

(ii) Descriptive model: It simply describes some aspects of a situation based on observation survey or other available data.

(iii) Predictive Model: such models make prediction regarding certain events

(iv) Deterministic model: Such modes assume conditions of complete certainly and perfect knowledge.

(v) Stochastic model: By such model it is possible to forecast a pattern of events based on which managerial decisions can be made. It is known as probabilistic model also.

(vi) Specific model: A model presents a system at some specific time is known as specific model.

(vii) Static model: If time factor is not considered in the model is called a static model.

(viii) Dynamic model: when a time is considered as one of the important variables and admits the impact of changes generated by time.

(ix) Mathematical and symbolical model: They are most abstract in nature. They employ a set of mathematical symbols to represent the relation between the components of the real system. These models are most general and precise.

(x) Simulation and heuristic model: These models have a specific mathematical structure and cannot be solved by purely using the tools and techniques of mathematics.

These models are used to explore alternative strategies were overlooked previously. Heuristic models do not claim to find the best solution to the problem.

E. General method for solving models

Usually the following three types of methods are used to solve OR models

(i) Analytic method: The tools of classical mathematics like:

Calculus, finite differences etc. are used for solving OR models. Solutions of various inventory models are obtained by this method. This method is also known as deductive methods.

(ii) Numerical methods: when classical methods fail due to complex city of constraints or variables then this method is adopted. This method is an iterative method or trial and error method. In this method, the iteration starts with a trial solution and a set of rules for improving it. The trial solution is then replace by the improved solution and the process is replaced until either no further improvement is possible or the cost of further calculation cannot be justified.

(iii) Monte Carlo method: This method involves the use of probability and concept of sampling theory. The steps associated with this method are as follows:

(a) Make sample observation and determine the probability distribution for the variables of intervals or the given model.

(b) Convert the probability distribution to cumulative distribution.

(c) Select a sequence of random numbers with the help of random number tables.

(d) With the help of the random numbers generated in the above steps determine the sequence of values of variables of interest.

(e) Fit an appropriate standard mathematical function to the values obtained in (d).

6. Brief outlines of OR models

A brief outline of some OR models is given below

(i) Distribution models: distribution models are concerned with the allocation of available resources so as to maximize profit or minimize loss subject to prescribed restrictions mathematical programming techniques are used to solve such models. The mathematical programming may be linear or non-linear.

(ii) Inventory models: This models are concerned with the determination of the economic order quantity and production intervals considering the factors like demand per unit time, shortage cost, held up cost, cost of ordering etc.

(iii) Queuing models: The problem related to queue of customer at a service counter or vehicles coming for service at a service station etc. such models are used. The points like: overage time spent in a queue, overage service time, traffic intensity are concerned in these models.

(iv) Competitive strategy models (Game theory models): These models are used to determine the strategy of decision maker under competition or conflict.

(v) Network models: This models are useful in large projects involving complexities and inter-dependencies of activities project evaluation and review technique (PERT) and critical path method (CPM) are used for planning, scheduling and controlling the project.

(vi) Job- sequencing models: These models are used to determine the best sequence of petering actions/jobs that optimize the efficiency measure of performance of the system.

(vii) Replacement models: These models determine the optimum replacement policy where some items or

machinery need replacement by a new one. Individual or group replacement policy can be used in case of the equipment that fails completely and instantaneously.

7. Mathematical modeling or formulation in linear programming problem (LPP)

A. Linear programming: It is a technique for determining an optimum schedule of independent activities in the view of available resources.

Here $a_1x_1 + a_2x_2 + a_3x_3 + \ldots\ldots a_nx_n,$ where $a_1, a_2 \ldots\ldots a_n,$ are constants and $x_1, x_2, x_3 \ldots\ldots x_n,$ are variables.

B. Objective function: A linear function of variables which is to be maximize or minimize is called objective function.

C. Constraints: A given set of equations and inequalities based on the given variables involved in the problem is called constraints or restrictions.

D. General procedure for mathematical formulation of LPP:

(i) Study the given situation to find the key decision to be made.

(ii) Identify the variables involved and denoted then by symbols $x_j,\ j = 1,2,3\ldots\ldots\ldots$

(iii) State the feasible alternatives, which are generally $x_j \geq 0 \text{ for all, } j$

(iv) Identify the constraints involved in the problem and express them as linear equation or inequality.

(v) Identify the objective function express it as a linear function of the decision variable.

8. Example with Solution

Example 1. Production Allocation problem:

A manufacturing company has four machine centers-machining, grinding, assembling and painting to produce four products A, B, C and D. The available numbers of hours per month in each of their centers are 250, 80, 650 and 400 hours respectively. The numbers of hours required by each product in each of these centers are given below:

	Products			
Types	A	B	C	D
Machining	1	1.5	3	1
Grinding	2	2	5	2
Assembling	5	4	6	6
Painting	2	3	3	3

The profit contribution for A, B, C and D is Rs. 20, Rs. 50, Rs.25, and Rs.40 respectively. Assuming there is enough demand for these products, how much of each product the company manufactures to maximize the total profit?

Solution: The given data we summarized as follows:

	Products				Available Time(in hours)
Types	A	B	C	D	
Machining	1	1.5	3	1	250
Grinding	2	2	5	2	80
Assembling	5	4	6	6	650
Painting	2	3	3	3	400
Profit (in Rs.)	20	30	25	40	

Step: 1 Here we have to determine the number of each types of product to be produced per month.

Let, X_1 be the number of units of the product type A

X_2 be the number of units of the product by type B

X_3 be the number of units of the product by type C

X_4 be the number of units of the product by type D per month.

Step:2 Feasible alternatives are $X_i \geq 0, i = 1,2,3,4.$

Step:3 Objective function: Since the profit of type A is Rs. 20per product, $20X_1$ will be the profit on X_1 units by type A. Similarly, $30X_2$ will be the profit on X_2 units by type B. $25X_3$ will be the profit on X_3 units by type C. $40X_4$ will be the profit on X_4 units by type D.

Hence the objective function

Maximize profit $Z = 20 X_1+30X_2+25X_3+40X_4$

Step 4: Construction of constraint:

For one unit of product type A, machining require 1 hour of time so for X_1 units of type A, time require for machine is $1* X_1$, similarly, the time for machining for X_2 units by type B becomes $1.5* X_2$ and for the product of type C and D time for machining become respective 3 X_3 and X_4.

Thus total time for all the four type of product for machining becomes

$$X_1+1.5X_2+3X_3+X_4$$

But it should not be exceeded the available 250 hours

$$X_1+1.5X_2+3X_3+X_4 \leq 250 \quad \text{...(i)}$$

Similarly for Gradding the constraints becomes

$$2X_1+2X_2+5X_3+2X_4 \leq 80 \quad \text{...(ii)}$$

For Assembling

$$5X_1+4X_2+6X_3+6X_4 \leq 650 \quad \text{... (iii)}$$

And for painting

$$2X_1+3X_2+3X_3+3X_4 \leq 400 \quad \text{...(iv)}$$

The Linear Programming Problem, therefore, can be put in the following mathematical format:

Maximize profit $Z = 20 X_1+30X_2+25X_3+40X_4$

Subject to constraints,

$$X_1+1.5X_2+3X_3+X_4 \leq 250$$

$$2X_1+2X_2+5X_3+2X_4 \leq 80$$

$$5X_1+4X_2+6X_3+6X_4 \leq 650$$

$$2X_1+3X_2+3X_3+3X_4 \leq 400$$

where; $X_1 \geq 0$, $X_2 \geq 0$, $X_3 \geq 0$ and $X_4 \geq 0$.

Example: 2 A cold drink company has two bottling plants located at two different places. Each plant produces three different drinks P, Q and R. The capacities of two plants in number of bottles per day are as follows:

	Product		
	P	Q	R
Plant-I	2000	1500	2500
Plant-II	1500	1500	5000

A market indicates that during any particular month there will be a demand of 25000 bottles of P, 20000 bottles of Q, and 40000 bottles of R. The operation cost per day of running plants are respectively, 700 Rs. and 500 Rs. How many days should the company run each plant during the month so that production cost is minimized while still meeting the market demand?

Let, company should run plant-I for X_1 days and plant-II for X_2 days

Therefore total cost for both the plants will be $700 X_1+500X_2$, which is to be maximize

Now determine the constraint

(i) For product P:

At plant I, 2000 units can be produced per day

Therefore in X_1 days 2000 X_1 units can be produced.

At plant II, 1000 units can be produced per day

Therefore in X_2 days 1000 X_2 units can be produced.

Therefore total production of type P becomes

$2000 X_1+1000X_2$.

But demand of product type P is of 25000 units

Therefore, $2000 X_1+1000X_2 \geq 25000$ ..(i)

(ii) For product Q:

At plant I, 1500 units can be produced per day

Therefore in X_1 days 1500 X_1 units can be produced.

At plant II, 1500 units can be produced per day

Therefore in X_2 days 1500 X_2 units can be produced

Therefore total production of type Q becomes

$1500 X_1+1500X_2$.

Therefore, $1500 X_1+1500X_2 \geq 20000$ ... (ii)

Similarly we can get constraints for product R as

$2500 X_1+5000X_2 \geq 40000$ ... (iii)

And $X_1 \geq 0$, $X_2 \geq 0$ ... (iv)

Hence the problem can be put in the following form:

Minimize $Z = 700 X_1+500X_2$

Subject to the constraints,

$2000 X_1+1000X_2 \geq 25000$

$1500 X_1+1500X_2 \geq 20000$

$2500 X_1+5000X_2 \geq 40000$

$X_1 \geq 0$, $X_2 \geq 0$.

9. Summary

Operations Research is a new approach used in the scientific study for problems solving under the given constraints.

It is useful to a decision maker to solve a complex problem which may not be possible to solve by any traditional methods. It is used in Finance, Purchasing, Marketing, production management, Research and Development, LIC and in many other areas. Three types of methods are used to solve OR models: Analytic method,

Numerical methods and Monte Carlo methods