Subject: Business Economics Course: B.A., 2nd Semester, Undergraduate.

Paper No: 202 Paper Title: Mathematics for Business Economics

Unit No.: 5 (Five) Title: Functions of two variables

Lecture No: 2 (Two) Title: Some More Aspects of Partial Differentiation

Glossary

Chain Rule: Let $z = f(y)$, where y is a function of another variable x i.e. $y = g(x)$ then the derivative of z with respect to x is equal to the derivative of z with respect to y, times the derivative of y with respect to x written symbolically as $\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx} = f'(y) \cdot g'(x)$. This rule is called Chain Rule.

Convex Combination: For any two points u, v of R^n, its Convex Combination is given by $tu + (1-t)v$ for all $0 \le t \le 1$.

Convex Sets: A subset S of points of R^n is said to be a Convex Set if for every two points in set S, the line segment joining these two points lies entirely in S.

Critical Point: Refer the definition of Stationary Point.

Euler's Theorem: Suppose $f : R^2 \to R$ is a homogeneous function of degree n and if f has continuous partial derivatives, then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = n\, f$

Explicitly Defined Function: A function of the type $y = f(x)$ where y is explicitly written in terms of x is called an Explicitly Defined Function.

Extrema: Either Maxima or Minima.

Global Maxima: Refer the definition of Maxima.

Global Minima: Refer the definition of Minima.

Implicitly Defined Functions: A function which is written in the form $F(x, y) = 0$ such that the explicit form of y in terms of x is not known is called an Implicitly Defined Function.

Local Maxima: Refer the definition of Maxima.

Local Minima: Refer the definition of Minima.

Maxima: Maxima of a function is the largest value of the function, either within a given range (the Local Maxima) or on the entire domain of a function (the Global or Absolute Maxima).

Minima: Minima of a function is the smallest value of the function, either within a given range (the Local Minima) or on the entire domain of a function (the Global or Absolute Minima).

Saddle Point: A point in the domain of a function of two independent variables is called Saddle Point if it is a stationary point but not a local extremum point.

i.e. $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$ and $\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} < \left(\frac{\partial^2 f}{\partial x \partial y} \right)^2$

Stationary Point of a Surface: A stationary point or a critical point of a differentiable function $f : A \rightarrow R,\ A \subset R^2$ of two variables is a point $a \in A$ where all the partial derivatives of f vanish.

Young's Theorem: Suppose f is a real valued function of two independent variables and if f has continuous second order partial derivatives at (x_0, y_0) in its domain, then $\frac{\partial^2 f}{\partial x \partial y}(x_0, y_0) = \frac{\partial^2 f}{\partial y \partial x}(x_0, y_0)$.