[Frequently Asked Questions]

[Concept of Level set & Partial Differentiation]

Subject:	Business Economics
Course:	B.A., 2nd Semester, Undergraduate
Paper No. & Title:	Paper – 202 (Two Zero two) Mathematics for Business Economics
Unit No. & Title:	Unit - 5 Functions of two variables
Lecture No. & Title:	1(One): Concept of Level set & Partial Differentiation

Frequently Asked Questions

1. **Give an example of a discontinuous function whose partial derivatives exist.**

Sol: $f: R^2 \to R$ defined as $f(x,y)=\begin{cases} \dfrac{xy}{x^2+y^2} & ,\ (x,y)\neq(0,0) \\ 0 & ,\ (x,y)=(0,0) \end{cases}$

$$\lim_{(x,y)\to(0,0)} f(x,y)=\lim_{\substack{y=mx\\x\to0}} f(x,y)=\lim_{x\to0} f(x,mx)=\lim_{x\to0}\frac{x(mx)}{x^2+(mx)^2}=\frac{m}{1+m^2}\neq 0 = f(0,0)$$

Therefore f is discontinuous at $(0,0)$.

Also,

$$\frac{\partial f}{\partial x}(0,0)=\lim_{h\to0}\frac{f(0+h,\,0)-f(0,0)}{h}=\lim_{h\to0}\frac{f(h,\,0)-f(0,0)}{h}=\lim_{h\to0}\frac{0-0}{h}=0$$

$$\frac{\partial f}{\partial y}(0,0)=\lim_{k\to0}\frac{f(0,\,0+k)-f(0,0)}{k}=\lim_{k\to0}\frac{f(0,\,k)-f(0,0)}{k}=\lim_{k\to0}\frac{0-0}{k}=0$$

Therefore the partial derivatives of first order exist at $(0,0)$ for f .

2. **Give an example of a continuous function whose partial derivatives don't exist.**

Sol: $f: R^2 \to R$ defined as $f(x,y)=|x+y|$.

$f(x,y)=|x+y|$ Is continuous function at $(0,0)$ and but its partial derivatives don't exist.

3. **Give an example of a no-where continuous function.**

Sol: $f: R \to R$ defined by $f(x)=\begin{cases} 1 & ,\ x\in Q \\ -1 & ,\ otherwise \end{cases}$.

This function is not continuous at any rational point α , as the limit taken on function $f(x)$ along a sequence of irrationals converging to α is −1 and $f(\alpha)=1$.

This function is not continuous at any irrational point α , as the limit taken on function $f(x)$ along a sequence of rationals converging to α is 1 and $f(\alpha)=-1$.

Hence the above function is no-where continuous on its domain R.

4. Give an example of a function which is continuous exactly at one point.

Sol: $f:R \to R$ defined by $f(x)=\begin{cases} x \ , & x \in Q \\ -x \ , & otherwise \end{cases}$.

This function is continuous only at $x=0$, as the limit taken on function $f(x)$ along a sequence of rational converging to 0 is 0 and also limit taken on function $f(x)$ along a sequence of irrationals converging to 0 is 0.

Further, the function is not continuous at any non-zero rational α because the limit taken on function $f(x)$ along a sequence of irrationals converging to α is $-\alpha$ and $f(\alpha)=\alpha$.

Also, the function is not continuous at any non-zero irrational α because the limit taken on function $f(x)$ along a sequence of rational converging to α is α and $f(\alpha)=-\alpha$.

5. Give an example of a bounded monotonically increasing function.

Sol: $f:[2, 4] \to R$ defined as $f(x)=x^3$.

$f(x)=x^3$ Is a continuous function and the domain of $f(x)=x^3$ here is a closed set.

Further, the image of a closed and bounded interval under a continuous function is closed and bounded.

Therefore $f(x)=x^3$ is a bounded function.

6. Prove: If a function $f:R \to R$ is monotonically increasing in its domain then $-f$ is monotonically decreasing in its domain.

Sol: Consider a function f to be monotonically increasing in its domain. Therefore $\frac{df}{dx} > 0$.

This implies that $-\frac{df}{dx} < 0 \quad \Rightarrow \frac{d}{dx}(-f) < 0$. This implies that $-f$ is monotonically decreasing function.

7. If $f(x,y)$ and $g(x,y)$ are functions defined on common domain and whose first order partial derivatives exist and are continuous at a point (x_0, y_0), then $\frac{f}{g}$ has continuous first order partial derivatives. True or False?

Sol:

False.

It is true only if further $g(x_0, y_0) \neq 0$.

8. What is the use of First Derivative Test?

Sol: First Derivative Test can be used to identify the region where the function increases or decreases.

The first derivative test states that a function is monotonic in an interval if its derivative does not change sign in that interval.

i.e. If the first derivative is positive in an interval then the function is strictly increasing in that interval and if the first derivative is negative in an interval then the function is strictly decreasing in that interval.

9. What is the use of Second Derivative Test?

Sol:

Second Derivative Test can be used to identify the points of maxima or minima of a function.

Further it also helps in locating the points of inflexion i.e. the points where the function changes its concavity.