ASSIGNMENT

1. Sketch the following sets in the plane.
   i. $\{ (x, y): 2x + y \le 4,\ x \ge 0 \ and \ \ y \ge 0 \}$
   ii. $\{ (x, y): 4x^2 + y^2 \le 4 \}$
2. Sketch some level curves of the following functions of two variables and describe their topography.
   a. $f(x, y) = 2x - y$
   b. $g(x, y) = (2x - y)^2$
   c. $h(x, y) = 2x^2 + y^2$
3. Given $y = u^3 + \log_e u$, where $\log_e u = 5 - t^2$ and $t = e^x + x^2$, find $\frac{dy}{dx}$ by using the chain rule.
4. Prove or disprove: The function $f(x) = x^3 - 3x^2 + 9x + 15$ is a monotonically increasing function.
5. Prove or disprove: The function $f(x, y) = 2x + y + 3\sqrt{xy}$ is a homogeneous function of two variables.
6. Prove or disprove: The function $f(x, y) = \frac{x^2 + y^2}{x - y}$ is a homogeneous function of two variables.
7. Compute all the partial derivatives up to second order of $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & , (x, y) \ne (0,0) \\ 0 \ , & (x, y) \ne (0,0) \end{cases}$ at point $(x, y) \ne (0,0)$.
8. Compute the first order partial derivatives of $f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2} & , (x, y) \ne (0,0) \\ 0 \ , & (x, y) \ne (0,0) \end{cases}$ at point $(x, y) = (0,0)$.