[Glossary]

Manipulations of Matrices and Determinants

Subject:	Business Economics
Course:	B. A. (Hons.), 2nd Semester, Undergraduate
Paper No. & Title:	Paper – 202 Mathematics for Business Economics
Unit No. & Title:	Unit – 4 Linear Algebra
Lecture No. & Title:	Lecture – 2 Manipulations of Matrices and Determinants

Glossary

Cancellation law for multiplication and addition of matrices:

Cancellation law for multiplication means:

If $AB = AD$ then $B = D$. This is false for Matrix Multiplicatin.

Cancellation law for addition means:

If $A + B = A + D$ then $B = D$. This is true for Matrix addition.

Determinant of a square matrix

The determinant of a $n \times n$ square matrix $\boldsymbol{A} = [\boldsymbol{a}_{ij}]$ is defined by:

$|\boldsymbol{A}| = \sum_{i=1}^{n} \boldsymbol{a}_{ij} \boldsymbol{C}_{ij}$ where $\boldsymbol{C}_{ij}$ is the cofactor of $\boldsymbol{a}_{ij}$ defined by

$\boldsymbol{C}_{ij} = (-\mathbf{1})^{i+j} M_{ij}$ where M_{ij} is the minor of matrix $\boldsymbol{A}$ i.e. the determinant of the $(n-1) \times (n-\mathbf{1})$ matrix formed by suppressing ith row and the jth column of matrix $\boldsymbol{A}$**.**

Adjoint of a square matrix:

Adjoint of a $n \times n$ square matrix $\boldsymbol{A} = [\boldsymbol{a}_{ij}]$, is defined and denoted by:

$Adj(A) = [C_{ij}]^T$

where $C_{ij} = (-\mathbf{1})^{i+j} M_{ij}$ and M_{ij} is the minor that is the determinant of $(n-1) \times (n-1)$ matrix obtained by suppressing the ith row and jth column of $\boldsymbol{A} = [\boldsymbol{a}_{ij}]$

Inverse of a square matrix:

Inverse of a □$\times n$ square matrix $\boldsymbol{A}$ is another square matrix $\boldsymbol{B}$ such that $\boldsymbol{AB} = \boldsymbol{BA} = \boldsymbol{I}$. It is not necessary that such a matrix $\boldsymbol{B}$

exists. If it exsits it is denoted by A^{-1}, and in this case we say that matrix A is invertible. Thus when A is invertible we have
$AA^{-1} = A^{-1}A = I$

Row rank of a matrix:

Suppose $A = [a_{ij}]$, is a $m \times n$ matrix. We define the row-rank of a A as the maximum number of linearly independent row vectors of $= [a_{ij}]$.

Column rank of matrix:

Column-rank of $A = [a_{ij}]$ is the maximum number of linearly independent column-vectors of A.

Rank of a matrix:

Suppose $A = [a_{ij}]$, is a $m \times n$ matrix. We define the row-rank of a A as the maximum number of linearly independent row vectors of $= [a_{ij}]$. Similarly column-rank of A is the maximum number of linearly independent column-vectors of A. It can be proved that for any matrix A the row-rank and the column-rank of A are equal. This common number is called the rank of A and we shall denote it by rank(A). It is clear that:
$rank(A) \leq min\{m, n\}$

System of simultaneous linear equations:

When there are m linear equations in n variables they are written as:

$a_{11}x_1 + a_{12}x_2 + \cdots a_{1n}x_n = b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots a_{2n}x_n = b_2$

$\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots a_{mn}x_n = b_m$$

Here a_{ij} , $i = 1,2 \ldots m \; and \; j = 1,2, \ldots n$ and b_i are given constant numbers.

And we are supposed to find, if exist, all numbers $x_1, x_2, \ldots, x_n$ which satisfy all the given m-equations simultaneously.

Matrix form of System of simultaneous linear equations:

When we have m equations in n variables, which can be written as:

$$\begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_m \end{bmatrix}$$

that is same as: $Ax = b$ where $A = [a_{ij}]$, $x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ \vdots \\ x_n \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ \vdots \\ b_m \end{bmatrix}$.

Consistent system of linear equations:

When there is at least one solution for the given system of linear equations, we say that the system is consistent.

Inconsistent system of linear equations:

When there is no solution for the given system of linear equations, we say that the system is inconsistent.