[Frequently Asked Questions]

Manipulations of Matrices and Determinants

Subject:	Business Economics
Course:	B. A. (Hons.), 2nd Semester, Undergraduate
Paper No. & Title:	Paper – 202 Mathematics for Business Economics
Unit No. & Title:	Unit – 4 Linear Algebra
Lecture No. & Title:	Lecture – 2 Manipulations of Matrices and Determinants

Frequently Asked Questions

Q1. Is matrix multiplication commutative?

A1. No. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $and\ B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ **then**

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix}$$

$$while\ BA = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5+18 & 10+24 \\ 7+24 & 14+32 \end{bmatrix}$$

Clearly it follows that in general $AB \neq BA$

Q2. Can it happen that $AB = 0$, but neither A nor B is a zero matrix?

A2. Yes.

Let $A = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$ then $A \neq 0$ also $B \neq 0$**.** But,

$$AB = \begin{bmatrix} 3 & 2 \\ 6 & 4 \end{bmatrix}\begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix} = \begin{bmatrix} 6-6 & -12+12 \\ 12-12 & -24+24 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Q3. Does the cancellation law for the Matrix Multiplication hold?

A3. No.

Suppose $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $D = \begin{bmatrix} 3 & 4 \\ 1 & 2 \end{bmatrix}$ then

$$AC = \begin{bmatrix} 1+3 & 2+4 \\ 1+3 & 2+4 \end{bmatrix}$$

$$AD = \begin{bmatrix} 3+1 & 4+2 \\ 3+1 & 4+2 \end{bmatrix}$$

Thus $AC = AD$**,** however $C \neq D$**.**

Q4. Can determinant be negative?

A4. Yes. $\begin{vmatrix} 1 & 2 \\ 2 & 1 \end{vmatrix} = -3$

Q5. Does the Inverse of every matrix exist?

A5. No. Determinant of an invertible matrix has to be non-zero. This is because $|AA^{-1}| = |A||A^{-1}| = |I| = 1$. So take any matrix whose determinant is 0. It will have no inverse. One can take $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Q6. Is every system of linear equation consistent?

A6. No. Consider

$2x + 3y = 6$

$4x + 6y = 9$

If there is a solution, then we get absurd result

$9 = 4x + 6y = 2(2x + 3y) = 12$

Q7. Can there be a linear system of equations having infinitely many solutions?

A7. Yes. Consider,

$x - y + z = 1$

$x + y + z = 1$

One immediately finds the infinite number of solutions as:

$\{(t, 0, 1 - t) \mid t \in \mathbb{R}\}$

Q8. Can a non-zero matrix have rank zero?

A8. No. Recall that the rank of a Matrix is the maximum order of a non-vanishing determinant that can be formed out of the given Matrix. So in this case rank is greater than or equal to 1.

Q9. Suppose in a $n \times n$ Matrix A there are two entries which are non-zero? Does it follow that rank of A also is two?

A9. No. One can consider. $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$.

Q10. Are determinants of A and A^{-1} reciprocals of each other?

A10. Yes. It follows from $\left|AA^{-1}\right| = |A|\left|A^{-1}\right| = |I| = 1$.