[Frequently Asked Questions]

Vector Spaces and Matrices

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Frequently Asked Questions

Q1.There is a zero $0 \in \mathbb{R}^n$ with the property that for all $x \in \mathbb{R}^n$, $x + 0 = 0 + x = x$. Is this 0 different when you change n?

A1. Yes. The zero of $\mathbb{R}^2$ is $(0,0)$ and that of $\mathbb{R}^3$ is $(0,0,0)$. We are denoting by a single notation 0. Its definite meaning should be clear from the context.

Q2.If a finite subset S of $\mathbb{R}^n$ contains the zero vector 0 of $\mathbb{R}^n$, can it be linearly independent?

A2. No. We can make a non-trivial linear combination of vectors from S to be a zero vector on scalar-multiplying to zero vector by any non-zero scalar and by real number zero to all remaining vectors of S.

Q3.Suppose T is a super set of a linearly dependent set S. That is S $\subseteq$ T. Is T linearly dependent?

A3. Yes. Since S is linearly dependent, a non-trivial linear combination of vectors from S is a zero vector. This non-trivial linear combination of vectors from S can be extended to a non-trivial linear combination of vectors from T by simply adding all zero scalar multiples of vectors from T-S.

Q4.Suppose T is a subset of a linearly independent subset S. Is T linearly independent?

A4. Yes. Consider any linear combination of vectors from T which is equal to a zero vector. Now this linear combination can be extended to a linear combination of vectors from S by adding zero scalar-multiples of vectors from S-T. Since S is linearly independent all scalars must be zero.

Q5.Can we have two bases for $\mathbb{R}^n$?

A5. Yes. $B_1 = \{ e^1, e^2, \dots, e^n\}$ and $B_2 = \{ 2e^1, 2\,e^2, \dots, 2e^n\}$ both are Bases for $\mathbb{R}^n$.

Q6.Suppose $n \times n$ square matrix J is having all entries equal to 1. Does J have the property AJ=JA=A for all $n \times n$ square matrices A?

A6. No. Clearly the first row, first column element in AJ will be the sum of all the elements of first row of A, which need not be a_{11}.

Q7.Suppose A and B are square matrices of the same type, is it true that $AB = BA$?

A7. No. If we take $\boldsymbol{A} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $\boldsymbol{and\ B} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$ **then**

$$\boldsymbol{AB} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix} = \begin{bmatrix} 5+14 & 6+16 \\ 15+28 & 18+32 \end{bmatrix}$$

$$\boldsymbol{while\ BA} = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 5+18 & 10+24 \\ 7+24 & 14+32 \end{bmatrix}.$$

Thus, $AB \neq BA$.

Q8.How is the phrase "orthogonal matrix" related to orthogonality of vectors?

A8. Any two distinct row vectors are orthogonal and any two distinct column vectors are also orthogonal.

Q9.Can vectors i.e. elements of $\mathbb{R}^n$ be regarded as matrices?

A9. Yes. Vectors i.e. elements of $\mathbb{R}^n$ can be regarded either as row vectors or as column vectors depending on the situation. When they are viewed as row vectors we can treat them as $1 \times n$ matrices. And when they are viewed as column vectors they can be treated as $n \times 1$ matrices.

Q10.If A and B are symmetric matrices then $AB = BA$?

A10. No. Suppose $A = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$. Both are symmetric matrices. But $AB = \begin{bmatrix} 3 & 3 \\ 1 & 2 \end{bmatrix}$ and $BA = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$, that is $AB \neq BA$.