[Frequently Asked Questions]

[Extreme Value & power series Representation of Function]

Subject: Business Economics

Course: B.A., 2nd Semester, Undergraduate

Paper No. & Title: Paper – 202 Mathematics for Business Economics

Unit No. & Title: Unit - 3 Single variable Differentiation

Lecture No. & Title: 1: Extreme Value & power series Representation of Function

Frequently Asked Questions

1. **Rules of Differentiation:**

Let f and g be any two differentiable functions at a , then the following rules are hold true.

(i) Addition Rule: $(f+g)'(a) = f'(a) + g'(a)$

(ii) Subtraction Rule: $(f-g)'(a) = f'(a) - g'(a)$

(iii) Scalar Multiplication Rule: $(\alpha \cdot f)'(a) = \alpha \cdot f'(a)$ for all $\alpha \in R$

(iv) Product Rule: $(f(x) \cdot g(x))'_{x=a} = f'(a) \cdot g(a) + f(a) \cdot g'(a)$

(v) Quotient Rule: $\left(\dfrac{f(x)}{g(x)}\right)'_{x=a} = \dfrac{g(a) \cdot f'(a) - f(a) \cdot g'(a)}{(g(a))^2}$ if $g(a) \neq 0$

(vi) Chain Rule: $(f \circ g)'(a) = f'(g(a)) \cdot g'(a)$ if function $(f \circ g)$ is well defined at a.

2. **Differentiation Formula for Some Regularly used Functions:**

(i) $\frac{d}{dx}(c) = 0$ where c is a constant

(ii) $\frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}}$

(iii) $\frac{d}{dx}\left(x^n\right) = n \cdot x^{n-1}$

(iv) $\frac{d}{dx}\left(e^x\right) = e^x$ and $\frac{d}{dx}\left(a^x\right) = a^x \log_e a$

(v) $\frac{d}{dx}(\log_e x) = \frac{1}{x}$ and $\frac{d}{dx}(\log_a x) = \frac{1}{x \log_e a}$

For more formulae, you may visit the link
https://en.wikipedia.org/wiki/Differentiation_rules

3. **Maclaurin's series for some regularly used function:**

(i) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \ldots$

(ii) $\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \frac{x^6}{6} + \ldots$

(iii) $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots$

(iv) $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots$

(v) $\tan x = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + \ldots$

4. **Is it possible to write power series expansion of every function?**
Ans: No. It can be written only for infinitely many times differentiable function.

5. **Give an example of a function whose Maclaurin's series expansion is not possible?**
Ans:

$$f(x) = \begin{cases} 1 & , x \le 0 \\ -1 & , x > 0 \end{cases}$$

6. **Give an example of a continuous function whose Maclaurin's series expansion is not possible?**
Ans:

$$f(x) = \begin{cases} x & , x \le 0 \\ -x & , x > 0 \end{cases}$$

7. **Is every stationary point of a function necessarily a point of maxima or minima?**
Ans: No.

Counter example:

$f(x) = x^3$

$\therefore\ f'(x) = 3x^2 \Rightarrow f'(0) = 0$ and $f''(x) = 6x \Rightarrow f''(0) = 0$.

$\therefore$ point $x = 0$ is a stationary point of $f(x) = x^3$, but it is neither point of maxima nor point of minima.

8. **Does there exist a function having infinitely many maxima and minima points?**
Ans: Yes.

Example:

$f(x) = \sin x$

$\therefore\ f'(x) = \cos x$ and $f''(x) = -\sin x$

On solving $f'(x) = \cos x = 0$ we get $x = (2k+1)\frac{\pi}{2}$, $\forall k \in Z$

Note that $f''\left((4k+1)\frac{\pi}{2}\right) = -\sin\left((4k+1)\frac{\pi}{2}\right) = -1 < 0$ &

$f''\left((4k+3)\frac{\pi}{2}\right) = -\sin\left((4k+3)\frac{\pi}{2}\right) = 1 > 0$

$\therefore$ The infinite set $\left\{(4k+1)\frac{\pi}{2} \ : k \in Z\right\}$ is the set of all maxima of $f(x) = \sin x$ and the infinite set $\left\{(4k+3)\frac{\pi}{2} \ : k \in Z\right\}$ is the set of all minima of $f(x) = \sin x$.