[Glossary]

Limit, Continuity & Differentiability of a real valued function of a real variable

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Glossary

Linear functions

A linear function is a function of the form $y = f(x) = mx + c$.

Polynomial functions

A real valued polynomial function is a function of the form $y = p(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ where $a_i's$ are real constants and n is a positive integer or zero.

Roots or zeroes of polynomials

For the polynomial $y = p(x)$, if there exists a real number c such that $y = p(c) = 0$, then we say c is a root or zero of the polynomial $p(x)$.

Rational functions

A real valued rational function is a function of the form $y = R(x) = \frac{p(x)}{q(x)}$ Where $p(x)$ and $q(x)$ are real valued polynomials in x. the domain of the rational function $R(x)$ is the set of all x where $q(x) \neq 0$.

Exponential function

An exponential function is a function of the form $y = f(x) = b^x$ defined for all $x \in \mathbb{R}$ and where b is some positive number different from 1.

Logarithmic function

As the exponential function $y = f(x) = b^x$ is a one-one function from $\mathbb{R}$ onto the set of positive reals, its inverse is a function from the set of positive reals onto $\mathbb{R}$. This function is called the logarithmic or in short log function with base b. The notation used for this function is $f^{-1}(y) = \log_b y$. Thus if y is any positive number then $f^{-1}(y) = \log_b y = x$ means that $b^x = y$.

Natural Logarithmic function

A natural logarithmic function is a logarithmic function with base as the number e.

Interval

An interval is a non-empty set of real numbers say I, such that if $x, y \in I$ and $x < z < y$, then $z \in I$.

Open Interval

An interval I is called an open interval if for every $x \in I$, there exist points $r, s \in I$ such that $r < x < s$.

Bounded and unbounded closed Interval

A bounded closed interval is a set of the type $\{x \in \mathbb{R}: a \leq x \leq b\}$ where a and b are real numbers. An unbounded closed interval is a set of the type $\{x \in \mathbb{R}: a \leq x\}$ or $\{x \in \mathbb{R}: x \leq b\}$

Limit of a real-valued function

Let $f(t)$ be a real-valued function which is defined on some open interval I containing a point x. We say that $f(t)$ has limit l as t tends to x or $f(t)$ has limit l at $t = x$, and write,

$\lim\limits_{t \to x} f(t) = l$, If for every $\epsilon > 0$ there exists $\delta > 0$, such that $|f(t) - l| < \epsilon$, whenever $t \in I \text{ and } 0 < |t - x| < \delta$.

Infinite valued limits

Let $f(t)$ be a real-valued function which is defined in some open interval I containing a point x. we say that $\lim_{t \to x} f(t) = \infty,$ if for every $M > 0$ there exists $\delta > 0$, such that $f(t) > M,$ whenever $t \in I \text{ and } 0 < |t - x| < \delta.$

On the other hand if for every $M > 0$ there exists $\delta > 0$, such that $f(t) < -M,$ whenever $t \in I \text{ and } 0 < |t - x| < \delta$ then we say that

$$\lim_{t \to x} f(t) = -\infty.$$

Limits at infinity

Let $f(t)$ be a real-valued function which is defined in some open interval of the form (a, ∞) and let $l \in \mathbb{R}$. Then we say that

$$\lim_{t \to \infty} f(t) = l,$$

if for every $\epsilon > 0$ there exists $M > 0$ such that $|f(t) - l| < \epsilon,$ whenever $t \in (a, \infty) \text{ and } t > M.$ Similarly if $f(t)$ is defined in some open interval of the form $(-\infty, b)$ then we say that $\lim_{t \to -\infty} f(t) = l$, if for every $\epsilon > 0$ there exists $M > 0$ such that $|f(t) - l| < \epsilon,$ whenever $t \in (-\infty, b) \text{ and } t < -M.$

Continuity of a real-valued function at a point

Let $f(t)$ be a real-valued function defined on some subset D of $\mathbb{R}$. We say that $f(t)$ is continuous at the point $x \in D$ if the domain set D contains an open interval containing the point $x,$ and further

$$\lim_{t \to x} f(t) = f(x).$$

Differentiability of a real-valued function at a point

Let $f(t)$ be a real-valued function defined on some subset D of $\mathbb{R}$. We say that $f(t)$ is differentiable at the point $c \in D$ if the domain set D contains an open interval containing the point c, and the limit

$$\lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$ Exists.