[Academic Script]

Social Welfare Function, Welfare Maximization, Fair Allocation, Envy and Equity, Arrow's Impossibility Theorem

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Academic Script

1. Introduction

'Welfare' meaning satisfactory state, health, prosperity, wellbeing, usually of a person or a society. In economics, we deal with person's utility or satisfaction, which depends upon the quality and quantity of good and services possessed by him. Thus in broad sense welfare depends upon the satisfaction level of all its consumers. While the study of optimal allocation of resources and their effect on social welfare is known as welfare economics.

The concept of Social welfare function, was propounded by, "A. Bergson" in his article "A Reformulation of certain aspects of Welfare Economics' in 1938. Prior to this, Both classical as well as neo- classical economists, has laid the framework of the concept based on cardinal measurability of utility and interpersonal comparison. As per Pareto, social welfare, can only be maximised if various marginal conditions of production, distribution and allocation of resources among products are satisfied.

While, Kaldor-Hicks-Scitovsky's Compensation principle gave a value free objective criterion based on ordinal concept of utility.

Here we are going to discuss various concepts of Welfare economics.

Social Welfare Function:

A social welfare function is a function that ranks different social states as more or less desirable or even indifferent for every pair of social states.

Concept of Social welfare function is explained by, various economists.

Here we have taken few.

The Classical Social Welfare Function: At first, Social welfare function was put forward by Bentham, Pigou and Marshall. 'According to them, social welfare is the sum of cardinal utilities obtained by all members of a society. Mathematically,

$$W=U1+U2+\ldots\ldots\ldots+Un$$

Where, W denote social welfare, U1, U2, U3 etc. represents the cardinal utilities of the individual members of the society.

The main aim of the society is to maximise social welfare, ie. The aggregate of the utilities of the individuals of the society. In classical welfare function, as per further assumption, the law of diminishing marginal utility applies to money income. Hence, maximum social welfare, can be achieved if income is so distributed that marginal utility of income is equal for all individuals in the society. Another assumption is that various individuals have the same tastes and thus, same capacity for satisfaction with the result that their utility functions are alike.

Thus, according to classical welfare function, maximisation of social welfare, is achieved only with equal distribution of income. But the concept of Classical welfare function is criticised by Modern economists because as per them, utility is ordinal concept and cannot be measured cardinally. While, ethical assumption of giving same weightage to all was also not accepted.

Pareto Social Welfare Function: According to Pareto's welfare function, maximum social welfare, is attained in an organisation, when one individual can be made better off without any one being worse off. This social welfare maximisation is known as 'Pareto optimality' or 'economic efficiency'.

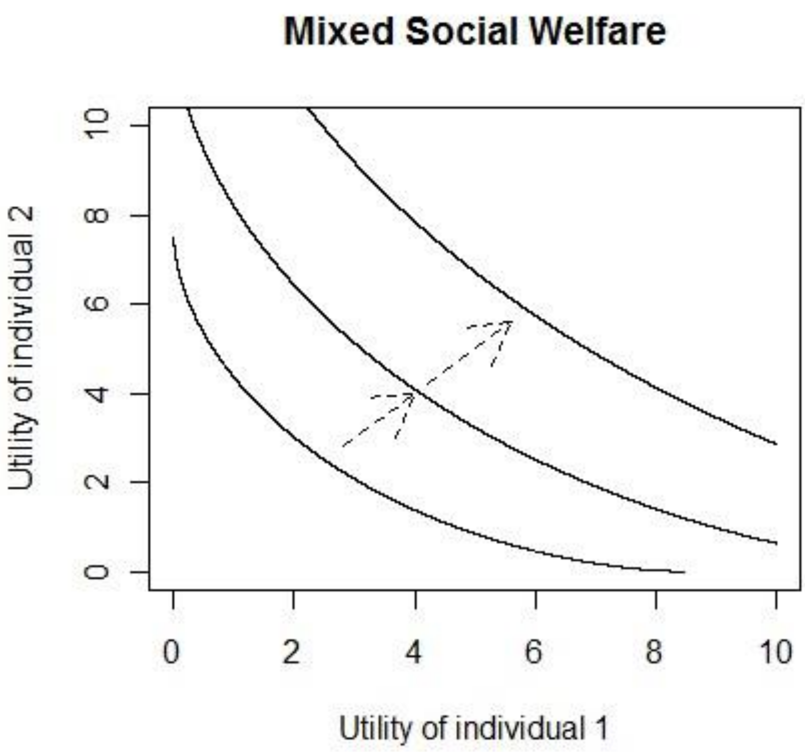
This was criticised on the basis that it is of limited operational significance, because with reorganisation and emergence of new economic policies some people become better off and other worse off.

And Bergson's Samuelson Social Welfare Function:

As per A. Bergson, in an ordinal index of society's welfare. Social Welfare is the function of utility levels of all individuals of the society. Thus, it is represented as:

W= f(u1,u2,u3,……..un)

Where, u1, u2, u3…. are utility index of individuals. Like indifference curve welfare functions can also have lower or higher levels. Movement along the social welfare curve makes the individual better off or worse off.



Construction of social welfare function is quite difficult, but not impossible. It could be considered by comparing deservedness of two individuals or could also be constructed by considering democracy through voting.

Compensation criteria: As per Kaldor's welfare criterion, if any change in economic organisation or policy makes some people better off and others worse off, then that change can increase social welfare only in case, gainers could compensate the losers, and still being better off.

Prof Hicks supported Kaldor's view and together they framed the Compensation principle, which states that "If A and B are two individuals, and A is better off, while B is worse off, then in any case if A contributes to B for its betterment, without losing anything", then this criterion is known as Compensation criterion.

This can also be explained with the help of a diagram, Here in this diagram we can see that if we move downwards on the curve DE, utility of A increases and that of B decreases. While, if we move upwards towards utility curve ED, utility of B increases and that of A decreases

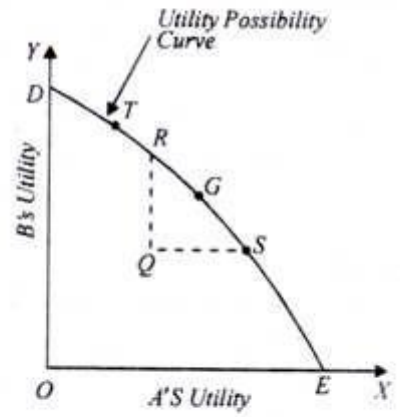


Fig. 41.1. *Kaldor-Hicks Criterion Explained with Utility Possibility Curve*

New Fig

Most of the economists believe that a movement is desirable only if no one is worse off and at least one person is better off. But practically speaking most changes would result in a reduction of someone's utility until and unless some compensation is provided.

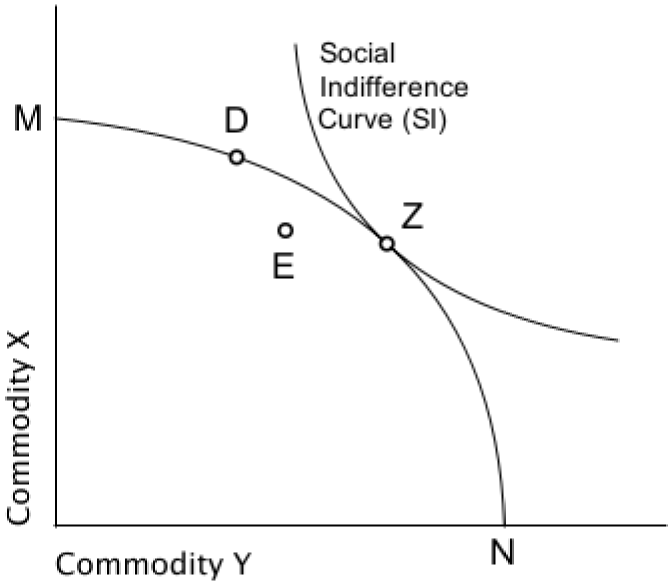
Three important criterion on this discussion are:

1. **The Kaldor's Criterion:** Allocation A is socially preferable to B if those who gain from A could compensate the losers.
2. **The Hicks Criterion:** Allocation A is socially preferable to B if those who would lose from A could not profitably bribe the gainers into not making the change from B to A.
3. **The Scitovsky Double Criterion:** Allocation of A is socially preferably if the gainers could bribe the gainers into accepting the change and simultaneously the losers could not bribe the gainers into not making the change.

This criterion implies interpersonal comparisons of utility, which are, expressed in terms of gains and losses of welfare in monetary terms. The criterion assumes that gainers and losers truthfully reveals their gains and losses. This criterion fails if they do not reveal, or either go for bargaining.

2. Welfare Maximisation

In welfare economics, we do not confine our self to individual welfare but go beyond that. An individual's welfare at anytime, is measured as the amount of satisfaction that he enjoys at that time. An individual always tries his best to maximize his satisfaction. An individual welfare is a function of so many economic and non-economic variables.



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As discussed, social welfare function is a combination of n number of utility functions, an envelope of these utility functions in a curve forms grand utility frontier. Each point on this curve represents an efficient allocation of resources i.e. it is Pareto efficient in factor allocation, production efficient and also consumption efficient . Thus point Z is the point where MRT= MRS

To understand welfare maximisation here in this figure we can see social indifference curve is tangent to Grand utility frontier at point Z. This point as discussed earlier is point of constrained bliss. Social welfare is maximum only at a point where both production efficiency and consumption efficiency coincides (i.e. MRT=MRS).

Here MN is the Grand utility frontier and curve I is the social indifference curve. Point Z is the point of constrained bliss. Thus Social welfare is maximum at this point 'Z'

Let us understand this in detail.

To construct a social welfare function, society must make interpersonal comparisons of different utilities. In order to get a social welfare function, society must compare the two individuals. For this purpose, a compensating criterion is developed.

By superimposing the social welfare function on grand utility frontier, we can determine the "point of constrained bliss".

Let us understand the concept of welfare maximisation with the help of diagram. In this figure, we can see that there are two utility indices A and B on x and Y-axis respectively. A grand utility possibility curve 'VV'' is superimposed on social indifference curves representing social welfare function to find unique optimum position of social welfare.

In this diagram there are four different social welfare curves W1, W2,W3, W4 and a curve VV' ie, Grand Utility Possibility curve is seen, which shows the various combinations of utilities received by individuals A and B. The point 'Q' where the grand utility possibility curve is tangent to W3, which is the highest attainable social welfare function, is called "point of constrained bliss".

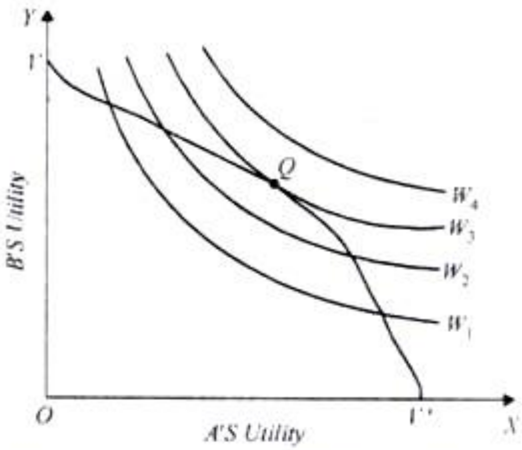


Fig. 42.2. *Social Welfare Function and Position of Constrained Bliss.*

Thus, from large number of Pareto optimum points on the grand utility possibility curve, we have unique optimum point 'Q' at which the social welfare is maximum. This point 'Q' shows both economic efficiency as well as equitable distribution made by the society. This point of constrained bliss represents the unique pattern of production of goods, distribution of goods and combination of factors employed to produce the goods.

3. Envy, Equity and Fair Allocation

Envy: If in a given allocation of fixed number of goods, between two individuals. If one prefers other's bundle more than our own then, we can say that, first person is envy of second.

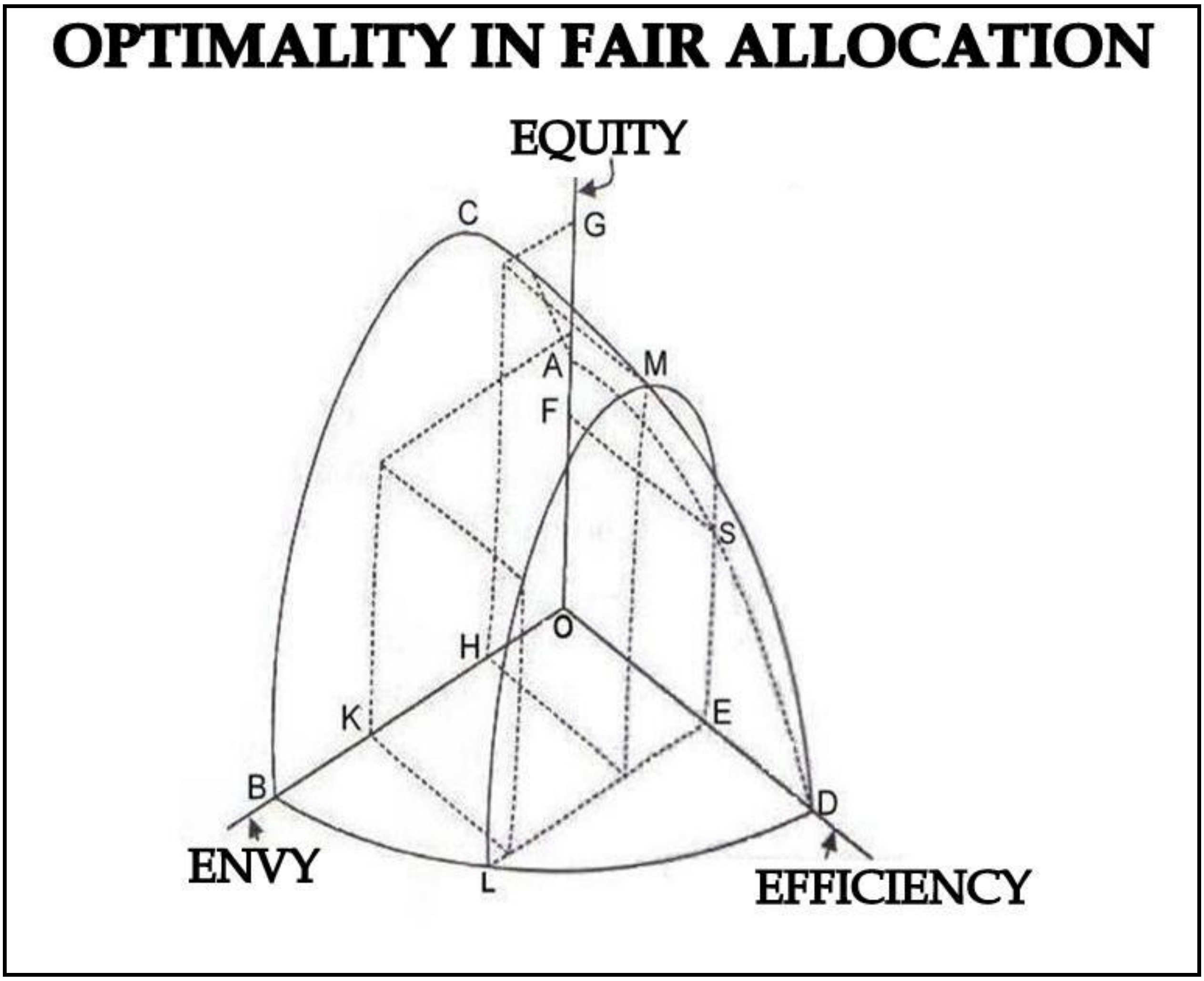
Equity: If in another allocation, preferences of all the individuals is indifferent of each other's choices. The allocation is said to be equitable.

Fair Allocation: Many economists tried to define the concept of fairness. Fair allocation is that allocation which is both Pareto efficient and Equitable. But practically Fair division is just a subjective concept.

As per the subjective theory of value, there is no objective measure of fairness. Thus, objective fairness is not possible. Fairness is a perception and differs from individual.

What is the most preferred by one individual? May not be preferred by another and vice versa. Let us take an example. Suppose there are three brothers A, B and C, and their father gives them a ballpoint pen each of same brand, But A prefers B's pen (colour preference) while B and C are happy with the allocation.

This allocation is not fair as A is not happy with the allocation, rather we can say A is envy of B. while, in another case if, A, B and C all are indifferent in their choices or we can say nonenvious, then the allocation is said to be equitable.



Thus, in the third case if the given allocation is both Pareto efficient and Equitable, The allocation is said to be Fair allocation.

if any change in economic organisation or policy makes some people better off and others worse off, then that change can increase social welfare only in case, gainers could compensate the losers, and still being better off.

This is better explained in this diagram, where there are trade-offs between envy and equity, Equity an efficiency, and efficiency and envy.

Meaning, that if an allocation is envy than it can never be equitable, or if it is just efficient it is not necessarily an equitable allocation, But if it is both than only it is said to be a fair allocation.

4. Arrow's Conditions for social welfare

Prof. Arrow has laid down five necessary conditions for social choices to satisfy in order to reflect the individual orderings. These are:

Condition 1: Transitivity or Consistency: Meaning, suppose an alternative A is socially preferred to alternative B and Alternative B is preferred to C, then C will not be preferred by alternative A.

Condition 2: Responsiveness to individual preferences: Meaning that social ranking must change in the same direction as with the choices of individual ranking.

Condition 3: Non- imposition condition: It implies that if no individual in the society prefers alternative A to B and any one or few of them prefers A to B then the society must prefer alternative A to B.

Condition 4: Non- Dictatorship: it states that if A must not be socially preferred to B only because any one of the individual in the society prefers A to B irrespective of the preferences of the others in the society.

Condition 5: Independence of Irrelevant alternatives: Meaning that social preference of A over B depends only on individual preferences of just these two and not on any other alternatives which is immediately not relevant.

All these five conditions reflects Arrows value judgement and seems to be quite reasonable set of conditions for making social choices in a free democratic society.

Thus, Arrow has revealed that it is impossible to make social welfare function on the basis of individual values that satisfy all the above conditions.

Arrows Impossibility Theorem:

After discussing social as well as individual value, Arrow proved his famous impossibility theorem.

According to Arrows Theorem, "If we exclude the possibility of interpersonal comparisons of utility, then the only method of passing through individual tastes to social preferences which will be satisfactory and which will be defined for a wide range of sets of individual origin are either imposed or dictatorial".

Thus, the only way to reach social choice is through voting i.e. the majority rule, but Arrow has revealed that consistent social choices cannot be made without violating the consistency or transitivity condition. The social choice on the basis of majority rule may be inconsistent even if individual preferences are consistent. Thus, out of all those five conditions discussed earlier, social welfare function based on individual preferences cannot be constructed in case of more than two alternatives.

Let us understand this theorem with the help of a table. In this table there are three individuals who constitute the society, have voted for three alternative social state X, y, and Z by writing 3 for most preferred, 2 for next preferred, and 1 for least preferred alternative. By looking into this table we can see that choices of all three individuals are contradictory.

Individuals	X	Y	Z
A	3	2	1
B	1	3	2
C	2	1	3

Thus, Arrow has proved that it is impossible to construct a social welfare function based on all individual preferences without involving inconsistency or non-contradictory social ranking.

Thus he has derived three consequences, in case of three alternatives X,Y and Z available to two individuals A and B:

1. Whenever the two individuals prefer X to Y then irrespective of any other alternative, society will prefer X to Y
2. If every individual in a group's preference for X and Y remains unchanged then, the societies preference among them remains unchanged.
3. If individuals A and B have exactly conflicting choices then, the society will be indifferent between X and Y.

Thus, in real life situation, Arrow has Shown, that Social Welfare Function based on all individual preferences, can never be constructed without violating at least one of these conditions.

5. Summary

Lets summarize at the end, Today, we have discussed the concept of welfare in the society, Social welfare function, Welfare maximisation, Compensation criterion, Envy, equity and Fair allocation concept, and Famous Arrows impossibility Theorem.