[Academic Script]

Probability

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Lecture No. & Title:	Lecture – 1 Probability

Academic Script

1. Introduction

In daily life we face many situations, where the results are not certain. With the help of probability theory one can make an attempt to measure the degree of uncertainty in the results of such situations. The theory of probability was originated from gambling but in current scenario it is useful in all fields of decision making.

Mathematicians Galileo, Pascal, Fermat, Bernoulli, De-Moivre, Bayes, Laplace etc. have given important contribution in developing the theory of probability whereas Chebyshev, Markoff, Liapounoff, Khintchine and Kolmogorov have made very valuable contribution to the modern theory of probability.

The probability theory is the mixture of set theory and counting. In counting mainly two things called permutations and combination play vital role. Among these items permutation and combination can be explained as

Permutation: If one thing can be done in m different ways and if with each way of doing it there are n ways of doing another thing, the two things together can be done in m x n ways is called permutation.

Combination: The number of ways of selecting r things out of n different things is called combination.

2. Terms used in probability Theory

Some terms are used for probability which are:

Random Experiment: An experiment which can result in any one of the several outcomes is called a random experiment. e.g. tossing of coin.

Sample Space: A set of representing all possible outcomes of a random experiment is called a sample space. e.g. If a coin is tossed then its sample space is U = {H, T}.

Events: The outcomes of an experiment are known as events. e.g. If a coin is tossed then two different events are head and tail.

Complementary Events: The compliment of a particular event is the aggregate of all the sample points of a sample space which do not belong to the particular event. e.g. If a coin is tossed then head is the complementary event of a tail.

Exhaustive events: If all possible outcomes of an experiment are considered, the outcomes are said to be exhaustive. e.g. If a coin is tossed then exhaustive events are head and tail.

Mutually exclusive events: Any events are said to be mutually exclusive, if they cannot occur to-gather. e.g. If a coin is tossed then head and tail are called mutually exclusive events.

Equally likely events: Events are said to be equally likely if chances of getting any outcome are same. e.g. If a coin is tossed then head and tail are equally likely events.

Favourable events: The members of a sample space which are favourable for an event are called favourable events. e.g. If a dice is tossed and a person is interested to get even number then the favourable events are {2, 4, 6}.

Independent events: Events are said to be independent if the happening of an event does not depend upon the happening or non-happening of the other events. e.g. If a coin is tossed then outcomes of Head (Tail) does not depend on the outcome Tail (Head), thus head and tail are independent events.

Conditional events: The event is called a conditional event if happening of the event is depending upon the happening of the other event or events. e.g. If on card is selected and there is chance that it is a king of red. Thus. Getting a king given that the selected card is red, the conditional event is denoted as $(\textbf{king} \mid \textbf{read})$. Conditional event is denoted by $A \mid B$, i.e. "A given B".

3. Definition of Probability

Based on these terms definition of probability is

Classical or mathematical definition: If an experiment can result in n exhaustive, mutually exclusive and equally likely ways, and if m of them are favourable to the happening of an event A, then the probability of happening of an event A is defined as the ratio of m to n.

$$P(A) = \frac{m}{n}$$

Axiomatic or modern definition: The modern concept of probability was introduced by a Russian mathematician Kolomogorov with the help of set theory.

If P(A) is a real number assigned to a subset A of a random sample S, then it is called the probability of an event A, provided P(A) satisfies the axioms which are

$0 \leq P(A) \leq 1$

$P(S) = 1$

If $A_1, A_2, A_3, \ldots$ is infinite sequence of disjoint events.

Then P (A) is called probability of an event A.

In probability theory some theorems are used their statements can be considered as

$P(A') = 1 - P(A)$

If A and B are any two events then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

For two events A and B

$$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

Where P (A) ≠ 0

$$i.e. P(B \cap A) = P(B/A).P(A)$$

Bayes' theorem: If event B can happen with mutually exclusive and exhaustive events A1, A2, A3,…, An and if the probabilities $P(A_1)$, $P(A_2)$, $P(A_3)$,…,$P(A_n)$ and also the conditional probabilities $P(B/A_1)$, $P(B/A_2)$, $P(B/A_3)$,…, $P(B/A_n)$ are known then the inverse probability of happening an event Ai under the condition that the event B has happened is given by

$$P(A_i / B) = \frac{P(A_i \cap B)}{P(B)}$$

$$= \frac{P(A_i / B)}{P(A_1 \cap B) + P(A_2 \cap B) + \ldots + P(A_n \cap B)}$$

$$= \frac{P(A_i)P(B / A_i)}{P(A_1).P(B / A_1) + P(A_2).P(B / A_2) + \ldots + P(A_n).P(B / A_n)}$$

4. Examples of Probability

Examples:

(Example of classical probability) Determine the probabilities of the following events in drawing card from a standard deck of 52 cards.

A seven

A black card

An ace or a king

A black two or a black three

A red face card

Solution:

There are four cards of "seven", therefore

P (seven) = 4/52 = 1/13

There are 26 black cards, therefore

P (black card) = 26/52 = 1/2

There are 4 ace and 4 king, therefore,

P (ace or king) = 8/52 = 2/13

There are 2 black colour cards of "two" and 2 black colour cards of "three", therefore

P (black two or three) = 4/52 = 1/13

There are 6 face cards of red colour, therefore

P (red face card) = 6/52 = 3/26

(Example based on counting) An urn contains 75 marbles: 35 are blue, and 25 of these blue marbles are swirled. The rest of them are red, and 30 of the red ones are swirled. The marbles that are not swirled are clear. What is the probability of drawing:

A blue marbles from the urn?

A clear marble from the urn?

A blue, swirled marble?

A red, clear marble?

A swirled marble?

Solution:

N=75

blue = 35

25 blue swirled

10 blue clear

red = 40

30 red swirled

10 red clear

a) P(blue) = 35/75

= 7/15

b) P(clear) =(10 + 10)/75

= 4/15

c) P(blue and swirled) = 25/75

= 1/3

d) P(red and clear) = 10/75

= 2/15

e) P(swirled) = (25 + 30)/75

= 11/15

(or 1 – P(clear) = 1 – 4/15 = 11/15)

(Example of joint probability) A bag contain 32 marbles: 4 are red, 9 are black, 12 are blue, 6 are yellow and 1 is purple. Marbles are drawn one at a time with replacement. What is the probability that

The second marble is yellow given that the first one was yellow?

The second marble is yellow given that the first one was black?

The third marbles is purple given both the first and second were purple?

Solution:

Here the first marble drawn was yellow. Now this marble is replaced in the same box. Then the second marble is drawn from the box. Here the result of the second draw does not depend on the outcome of the first draw, so probability of getting yellow marble at second draw is same as the probability of drawing a yellow marble (y) from the box.

Hence, P (Y)= 6/32

As discussed in the case (s), as an experiment is done by with replacement, there is no effect of the first draw on the outcome of the second draw, the probability of getting yellow marble at the second draw what ever be the outcome at the first draw is same as the probability of getting yellow marble at the first draw.

Hence required probability P (Y)= 6/32

Here also probability of getting purple marble at the third draw is same as the probability of getting purple marble at the first draw.

Here P (purple marble at the third draw)= 1/32

(Example of conditional probability) At a soup kitchen, a social worker gathers the following data. Of those visiting the kitchen, 59 percent are men, 32 percent are alcoholics and 21 percent are male alcoholics. What is the probability that a random male visitor to the kitchen is an alcoholic?

Solution:

P (alcoholic / male) = $\frac{P(\text{male and alcoholic})}{P(\text{male})}\ \frac{.21}{.59}$ = 0.356

(Example of conditional probability) During a study of auto accidents, The Highway Safety Council found that 60 percent of all accidents occur at night, 52 percent are alcohol related and 37 percent occur at night and alcohol-related.

What is the probability that an accident was alcohol-related, given that it occurred at night?

What is the probability that an accident occurred at night, given that it was alcohol-related?

Solution:

a) $P(A \mid N) = \frac{P(A \text{ and } N)}{P(N)} = \frac{.37}{.6} = 0.617$

b) $P(N \mid A) = \frac{P(A \text{ and } N)}{P(A)} = \frac{.37}{.52} = 0.712$

(Example of Bayes' theorem) A company is planning its company picnic. The only thing that will cancel the picnic is a thunderstorm. The Weather Service has predicted dry conditions with probability 0.2, moist condition with probability 0.45 and wet condition with probability 0.35. If the probability of a thunderstorm given dry condition is 0.3, given moist condition is 0.6 and given wet condition is 0.8, what is the probability of thunderstorm? If we know the picnic was indeed canceled, what is the probability moist conditions were in effect?

Event	P(Event)	P(Storm / Event)	P(Storm & Event)	P(Event / Storm)
Dry	0.20	0.30	0.06	0.06/0.61 = 0.0984
Moist	0.45	0.60	0.27	0.27/0.61 = 0.4426

Wet	0.35	0.80	0.28	0.28/0.61 = 0.4590
			P(Storm) = 0.61	

Solution:

The probability of a thunderstorm is 0.61. The probability of moist conditions, given that the picnic was cancelled (i.e., given that there was a thunderstorm) is 0.4426.

5. Summary

We have discussed probability as one of the important tools to get solution of daily life where we face many situations and where the results are not certain. With the help of probability theory one can make an attempt to measure the degree of uncertainty in the results of such situations. All distributions are based on probability theory. It is also heavily used for getting decision in business and economics.