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Index Numbers (Part – 2)

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1. Introduction

In Index Number – Part 1 we have talked about the different ways of constructing index numbers. As none of the index number formulae measures the price or quantity changes with perfection and have some bias, statisticians have devised a number of tests to choose the most appropriate formula in a given situation. These tests are termed as Tests of Adequacy.

2. Test of Accuracy

1. The following four tests are usually used for index numbers.

(a) Unit Test

(b) Time Reversal Test

(c) Factor Reversal Test

(d) Circular Test

(a) Unit Test: This test requires that the index number formula should be independent of the units in which the prices or quantities of various commodities are quoted. All the formulae discussed satisfy unit test. The index number which is based on simple aggregate of prices or quantities without attaching any weights to them,

i.e.

$$P_{01} = \frac{\sum p_1}{\sum p_0} \times 100 \quad \text{or} \quad Q_{01} = \frac{\sum q_1}{\sum q_0} \times 100$$

does not satisfy the unit test. This is because all the commodities do not have the same unit of measurement. Moreover some commodities are costly and others are cheap. Some are essential goods while others are luxury items. All this should be reflected in the weights attached to them. But in simple aggregate of prices and quantities index numbers weights are not considered, so they do not satisfy the unit test.

(b) Time Reversal Test: This test requires the index number formula to possess time consistency by working both forward and backward with respect to time. It means if we reverse the time subscripts of a price or quantity index the result should be the reciprocal of the original index.

i.e. $P_{01} \times P_{10} = 1$

This test is satisfied by simple aggregate index, Marshall-Edgeworth index and Fisher index. Laspeyres and Paasche indices do not satisfy this test, except when both are equal.

(c) Factor Reversal Test: This test requires that if the price and quantity indices are obtained for the same data, same base and current periods and using the same formula, then their product, without the factor 100, should give a true value ratio.

i.e.

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = V_{01}$$

where $\sum p_1 q_1$ and $\sum p_0 q_0$ denote the total value in the current and base year respectively. Fisher index is the only index satisfying this test, so it is also known as Fisher Ideal Index. If Laspeyres index is equal to Paasche index, then both these index numbers satisfy the factor reversal test.

(d) Circular Test: This is an extension of the time reversal test over several years and thus more searching than any other single time reversal test which it contains. It requires the index to work in circular manner which enables to find the index numbers from period to period without referring back to the original base each time. The test requires

$P_{01} \times P_{12} \times P_{23} \times \dots\dots\dots\dots\dots\dots\dots\dots\dots \times P_{n-1,n} \times P_{no} = 1$

By time reversal test $P_{0n} \times P_{no} = 1$

$$=> P_{no} = \frac{1}{P_{0n}}$$

$$=> P_{0n} = P_{01} \times P_{12} \times P_{23} \times \dots\dots\dots\dots\dots\dots\dots\dots\dots \times P_{n-1,n}$$

This test is satisfied by the aggregate index with fixed weights which is also called Kelly fixed base method.

$$P_{01}^{K} = \frac{\sum wp_1}{\sum wp_0}$$

where instead of using base period or current period quantities as weights, w is the weight from any representative period.

Example 1: For the given data show that Laspeyres and Paasche indices do not satisfy the Time Reversal test and the Factor Reversal test, but Fisher Ideal index satisfies both the tests.

Commodity	Base Year		Current Year	
	Price	Quantity	Price	Quantity
A	7.5	600	11.8	660
B	3.8	224	3.9	248
C	5.7	79	9.2	88
D	11.9	48	14.4	34
E	9.6	59	11.8	37

Solution:

Commodity	p_0	q_0	p_1	q_1	p_0q_0	p_0q_1	p_1q_0	p_1q_1
A	7.5	600	11.8	660	4500	4950	7080	7788
B	3.8	224	3.9	248	851.2	942.4	873.6	967.2
C	5.7	79	9.2	88	450.3	501.6	726.8	809.6
D	11.9	48	14.4	34	571.2	404.6	691.2	489.6

E	9.6	59	11.8	37	566.4	355.2	696.2	436.6
Total					6939.1	7153.8	10067.8	10491

Time Reversal Test: $P_{01} \times P_{10} = 1$

Laspeyres Price Index:

$$P_{01}^{La} \times P_{10}^{La} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_0 q_1}{\sum p_1 q_1} = \frac{10067.8}{6939.1} \times \frac{7153.8}{10491} = 0.9893 \neq 1$$

Paasche Price Index:

$$P_{01}^{Pa} \times P_{10}^{Pa} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0} = \frac{10491}{7153.8} \times \frac{6939.1}{10067.8} = 1.0107 \neq 1$$

Fisher Price Index:

$$P_{01}^{F} \times P_{10}^{F} = \sqrt{P_{01}^{La} \times P_{01}^{Pa}} \times \sqrt{P_{10}^{La} \times P_{10}^{Pa}} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times \sqrt{\frac{\sum p_0 q_1}{\sum p_1 q_1} \times \frac{\sum p_0 q_0}{\sum p_1 q_0}}$$

$$= \sqrt{\frac{10067.8}{6939.1} \times \frac{10491}{7153.8}} \times \sqrt{\frac{7153.8}{10491} \times \frac{6939.1}{10067.8}}$$

$$= 1$$

Thus Fisher index satisfies time reversal test and Laspeyres and Paasche indices do not satisfy the test.

Factor Reversal Test:

$$P_{01} \times Q_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = V_{01}$$

Laspeyres Index:

$$P_{01}^{La} \times Q_{01}^{La} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum q_1 p_0}{\sum q_0 p_0} = 1.4509 \times 1.0309 = 1.4958$$

$$V_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = 1.5119$$

$$=> P_{01}^{La} \times Q_{01}^{La} \neq V_{01}$$

Paasche Index:

$$P_{01}^{Pa} \times Q_{01}^{Pa} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times \frac{\sum q_1 p_1}{\sum q_0 p_1} = 1.4665 \times 1.0420 = 1.5281$$

$$V_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = 1.5119$$

$$=> P_{01}^{Pa} \times Q_{01}^{Pa} \neq V_{01}$$

Fisher Index:

$$P_{01}^{F} \times Q_{01}^{F} = \sqrt{P_{01}^{La} \times P_{01}^{Pa}} \times \sqrt{Q_{01}^{La} \times Q_{01}^{Pa}} = \sqrt{\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}} \times \sqrt{\frac{\sum q_1 p_0}{\sum q_0 p_0} \times \frac{\sum q_1 p_1}{\sum q_0 p_1}}$$

$$= \sqrt{\frac{(\sum p_1 q_1)^2}{(\sum p_0 q_0)^2}} = \frac{\sum p_1 q_1}{\sum p_0 q_0} = V_{01}$$

Thus Fisher index satisfies factor reversal test and Laspeyres and Paasche indices do not satisfy the test.

3. Base Shifting, Splicing and Deflating of Index Numbers

Base Shifting of Index Numbers: Every index number must have a starting point, a date or length of time from which changes can be measured. The base may be one day, the average of a year or the average of a series of years. To make meaningful and valid comparisons, the base year should be normal year of economic stability not too far distant from the given year. If we want to compare series of index numbers with different base periods, both the series must be expressed with a common base period. Here base shifting is required, which means the changing of the given base period of a series of index numbers and recasting them into a new series based on some new base period.

Splicing of Index Numbers : Splicing is an application of the principle of base shifting, by which two or more overlapping series of index numbers are combined to obtain a single continuous series.

Deflating of Index Numbers: Deflating is the process of eliminating the price effect from a given set of monetary values. Deflating of index numbers is desirable to determine real income and purchasing power of money in an economy which has inflationary trends.

$$Real\ Income = \frac{Nominal\ Income}{Price\ Index} \times 100$$

$$Purchasing\ Power = \frac{1}{Price\ Index} \times 100$$

Example 2: A firm in a certain industry has an index of material prices, based on movements in the prices of selected materials weighted by the quantities consumed in the base year. The price index series based on 1997=100, for the years 2007-2012 was

2007	2008	2009	2010	2011	2012
130.3	132.1	136.4	135.2	137.0	141.6

In 2012, the index was completely revised to take into account a change in the type of materials used. The new index, based on 2012=100, showed the values as

2012	2013	2014
100	116.3	119.4

(i) Splice the new index to the old, i.e., splice 'forward'.

(ii) Splice the old index to the new, i.e., splice 'backward'.

Solution: Splicing the new index to the old, i.e., splice 'forward'

Here the indices up to the years 2011 remains as it is and from the year 2012 they will change.

Year	Index (1997=100)	Year	Index(1997=100)
2007	130.3	2012	141.6 (100 for 2012=100)
2008	132.1	2013	$\frac{141.6}{100} \times 116.3 = 164.7$
2009	136.4	2014	$\frac{141.6}{100} \times 119.4 = 169.1$
2010	135.2		
2011	137.0		

(ii) Splicing the old index to the new, i.e., splice 'backward'

Here the indices from 2007 to 2011 will change, from 2012 they remains as it is.

Year	Index (1997=100)	Year	Index(1997=100)
2007	$\frac{100}{141.6} \times 130.3 = 92.0$	2012	100 (141.6 for 1997=100)
2008	$\frac{100}{141.6} \times 132.1 = 93.3$	2013	116.3
2009	$\frac{100}{141.6} \times 136.4 = 96.3$	2014	119.4
2010	$\frac{100}{141.6} \times 135.2 = 95.5$		
2011	$\frac{100}{141.6} \times 137 = 96.8$		

4. Construction of Real Indices

(a) Consumer Price Index

(b) BSE Index

(a) Consumer Price Index: Consumer price index number is a measure of change in the price level of a basket of goods and services purchased by households during any given period with respect to some fixed base period. It is based on retail prices and thus is also termed as Retail Price Index Number or Cost of Living Index Number.

The consumption patterns of various commodities differ widely from class to class, like poor, lower income group, high income group, labour class, industrial workers, agricultural workers and within same group from region to region, like town, city, rural area, urban area, planes and hills. To study the effect of rise or fall in the prices of various commodities on the purchasing power, the consumer price index numbers are constructed for different classes of people of the society and also for different geographical areas. The final overall index is calculated using the various indices and sub-indices. The consumer price index number is constructed by two methods.

Method 1: Aggregate Expenditure Method or Weighted Aggregate Method

In this method, quantities consumed in the base year are used as weights, which is nothing but Laspeyres price index.

$$Consumer\ Price\ Index = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

Method 2: Family Budget Method or Method of Weighted Relatives

In this method, the consumer price index is obtained on taking the weighted average of price-relatives, the weights being the values of the quantities consumed in the base year.

$$Consumer\ Price\ Index = \frac{\sum wP}{\sum w}$$

where

$$P = price\,relative = \frac{p_1}{p_0} \times 100$$

$$w = p_0 q_0$$

$$Consumer\ Price\ Index = \frac{\sum p_0 q_0 \left(\frac{p_1}{p_0}\right) \times 100}{\sum p_0 q_0} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

The consumer price index number obtained by both the methods is same. It is used to measure inflation by computing purchasing power of money. The consumer price index is also used to compute the real wages from the nominal wages.

$$Purchasing\ Power\ of\ Money = \frac{1}{Consumer\ Price\ Index} \times 100$$

$$Real\ Wages = \frac{Nominal\ Wages}{Consumer\ Price\ Index} \times 100$$

Example 3: The cost of living index number for 2008 with 2000 as base for different commodity groups: Food, Clothing, Fuel and Light, Rent and Miscellaneous are 540, 600, 450, 500 and 350 respectively with their weights in order in the ratio 14 : 2 : 3 : 1 : 5.

Obtain the overall cost of living index number. Suppose a person was earning Rs. 40,000 in 2000. What should be his salary in 2008 to maintain the same standard of living as in 2000?

Solution:

$$Cost\ of\ Living\ Index = \frac{\sum wI}{\sum w}$$

$$= \frac{\sum(14 \times 540 + 2 \times 600 + 3 \times 450 + 1 \times 500 + 5 \times 350)}{\sum(14 + 2 + 3 + 1 + 5)} = 494.4$$

$$Purchasing\ Power\ of\ Money\ in\ 2008 = \frac{1}{Cost\ of\ Living\ Index\ in\ 2008} \times 100$$

$$= \frac{1}{494.4} \times 100 = 0.202$$

To maintain the same standard of living as in 2000, the salary in 2008 should be

$$Salary\ in\ the\ year\ 2008$$

$$= \frac{Salary\ for\ the\ year\ 2000}{100} \times Price\ Index$$

$$= \frac{40000}{100} \times 494.4 = 1{,}97{,}760$$

(b) BSE Index: Bombay Stock Exchange Index is the weighted stock market index computed on the basis of free float capitalization method. Free float means the proportion of total issued shares of the company that are traded in the stock market. It is different from market capitalization of a company which is determined by multiplying the price of one share by the number of outstanding shares of the company. The BSE index shows the free float market value of 30 stocks in the current period relative to a base period. It considers 30 well established and financially sound companies representative of the various industrial sectors of the Indian economy that are listed on Bombay Stock Exchange. The base year of the BSE index is 1978-79 and the base value is taken as 100 on 1st April 1979.

5. Summary

The different index number formulae that measure price and quantity changes have some bias. So statisticians have devised a number of tests to choose the most appropriate formula in a given situation. These tests are

1. Unit Test
2. Time Reversal Test
3. Factor Reversal Test
4. Circular Test

The unit test requires that the index number formula should be independent of the units in which the prices and quantities of the various commodities are quoted. All the formulae, except simple aggregate index number satisfy the unit test. The time reversal test requires the index number formula to possess time consistency by working both forward and backward with respect to time. Simple aggregate index, Marshall-Edgeworth index and Fisher index satisfy the time reversal test. Factor reversal test requires that the product of price and quantity indices should give the true value. Fisher index is the only index that satisfies factor reversal test and so it is also known as Fisher Ideal Index. Circular test is an extension of the time reversal test over several years. Kelly fixed base index satisfy the circular test.

To compare series of index numbers with different base periods, the base of the series is changed to have a common base period. The process is termed as base shifting. The application of base shifting is in splicing, where two or more overlapping series of index numbers are combined to obtain single continuous series. Real income and purchasing power of money can be obtained by deflating of index numbers.

The consumer price index is a measure of change in the price level of a basket of goods and services purchased by households during any given period with respect to some fixed base period. It is used to compute real income and purchasing power of money to know the level of inflation. BSE index is the weighted stock market index computed on the basis of free float capitalization method by considering 30 well established and financially sound companies. These 30 companies are representative of the various industrial sectors of the Indian economy that are listed on Bombay Stock Exchange.