[Academic Script]

Index Numbers (Part – 1)

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1. Introduction

An index number is a measure of change in magnitude from one situation to another. The two situations may be two time periods, two regions of a country or two groups of individuals. In economics and finance, index number is a statistical measure, which reflects the relative changes in the level of a certain phenomenon, say, prices or wages in any given period, called the current period with respect to its value in some fixed period, called the base period. The reference base is expressed as having one selected situation as 100. Another year may have an index number, say, 138. This means that the magnitude in the second year is 138% of its level in the base year. The actual level is measured in neither of the years; only the change from one year to the other is given by the index. Here, the index number shows the increase of 38%. Therecan be indices for the data of volume of trade, industrial and agricultural production, employment, bank deposits and foreign exchange reserves.

Economic indices track economic health from different perspectives. Financial indices track the performance of selected large and powerful companies in order to evaluate and predict economic trends.

For economic and business study index numbers may be broadly classified into three categories.

i) Price Index Numbers

ii) Quantity Index Numbers

iii) Value Index Numbers

The price index number measures the general changes in the prices. The two specific types of price indices are whole sale price index numbers and retail price index numbers. Quantity index number studies the changes in the volume of goods produced, distributed and consumed. These indices can be computed for agricultural and industrial production, imports and exports. Value index number measures the change in the total value of production, such as indices of inventories, sales and profits.

Value = Price X Quantity

For constructing index numbers, select a basket of items that need to be focused for the study. All the items may not have same unit of measurement. For example, if we are constructing index numbers for the set of household commodities, such as cereals, milk, cloth, petrol and electronic items, all the commodities do not have same unit of measurement. Cereals may be quoted in Rs. per kilogram or quintal; liquids, like milk and petrol may be quoted in Rs. per liter; cloth may be quoted in Rs. per meter and so on. Some are costly goods and others are cheap. Some are essential goods while others are luxury items. So to balance their importance of use and to construct an index number which is unit-less, different authors have given different procedures of constructing index numbers. They have adopted different ways of assigning weights to the commodities.

2. Different Index Numbers

Consider n commodities. Let the given data be the prices and quantities for these n commodities. The data relate to specified groups of buyers or sellers in defined markets or geographical areas and for particular time periods. The prices and quantities of the n commodities in year 0 i.e., the base year, be denoted by p_{i0} andq_{i0}(i=1,2,…………,n). So we write $\sum_{i=1}^{n} p_{i0} q_{io} = \sum p_0\, q_0$and similarly other expressions.

There are some index numbers constructed by using different systems of weighting.

i. Laspeyres Price Index Number:

$$P_{01}^{La} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100$$

P_{01} - price index for the current year with respect to the base year

p_0 - price of the commodity in the base year

p_1 - price of the commodity in the current year

q_0- quantity of the commodity in the base year

In this index number, the base year quantities are taken as weights. This method of computing price index is also called Base Year Method.

ii. Paasche Price Index Number:

$$P_{01}^{Pa} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100$$

q_1- quantity of the commodity in the current year

Here the current year quantities are taken as weights.

iii. Laspeyres and Paasche Quantity Index Numbers

$$Q_{01}^{La} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100$$

$$Q_{01}^{Pa} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100$$

By convention year 0 is the earlier and year 1 is the later year. The comparison from year 0 to year 1 is called forward. Equally, the backward comparison is from year 1 to year 0. So $P_{01}$and $Q_{01}$are forward and P_{10} and Q_{10} are backward index numbers of price and quantity.

i.e.

$$P_{10}^{La} = \frac{\sum p_0 q_1}{\sum p_1 q_1} \times 100$$

$$Q_{10}^{La} = \frac{\sum q_0 p_1}{\sum q_1 p_1} \times 100$$

$$P_{10}^{Pa} = \frac{\sum p_0 q_0}{\sum p_1 q_0} \times 100$$

$$Q_{10}^{Pa} = \frac{\sum q_0 p_0}{\sum q_1 p_0} \times 100$$

Value Index: This measures the changes in the value of a group of commodities in the current period with respect to the base period.

$$V_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} \times 100$$

Example 1: From the given data construct the Laspeyres andPaasche price and quantity index numbers. Also compute the Value index number.

Base Year

CurrentYearCommoditiesUnits Price Expenditure Price Expenditure

per unit (Rs.) per unit (Rs.)

('00Rs.) ('00Rs.)

A litre2 40 5 75

B metre 4 16 8 40

C kg1 10 2 24

D sq. ft. 5 25 10 60

Solution: $Expenditure = Price \times Quantity$

$$=> Quantity = \frac{Expenditure}{Price}$$

Commodities	p_0	q_0	p_1	q_1	$p_0 q_0$	$p_1 q_1$	$p_0 q_1$	$p_1 q_0$
A	2	20	5	15	4075	30	100	
B	4	4	8	5	16	40	20	32
C	1	10	2	12	10	24	12	20
D	5	5	10	6	25	60	30	50

$$\sum p_0 q_0 = 91, \quad \sum p_1 q_1 = 199, \quad \sum p_0 q_1 = 92, \quad \sum p_1 q_0 = 202$$

$$P_{01}^{La} = \frac{\sum p_1 q_0}{\sum p_0 q_0} \times 100 = \frac{202}{91} \times 100 = 221.98$$

$$Q_{01}^{La} = \frac{\sum q_1 p_0}{\sum q_0 p_0} \times 100 = \frac{92}{91} \times 100 = 101.1$$

$$P_{01}^{Pa} = \frac{\sum p_1 q_1}{\sum p_0 q_1} \times 100 = \frac{199}{92} \times 100 = 216.3$$

$$Q_{01}^{Pa} = \frac{\sum q_1 p_1}{\sum q_0 p_1} \times 100 = \frac{199}{202} \times 100 = 98.5$$

$$V_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0} \times 100 = \frac{199}{91} \times 100 = 218.68$$

3. Relation Between Laspeyres and Paasche Index Numbers

i). Suppose

$$V_{01} = \frac{\sum p_1 q_1}{\sum p_0 q_0}$$

is the change in actual value from year 0 to year 1.

Paasche price index is V_{01} divided by the Laspeyres quantity index and the Paasche quantity index number similarly is V_{01}divided by the Laspeyres price index.

i.e.

$$P_{01}^{Pa} = \frac{V_{01}}{Q_{01}^{La}}$$

and

$$Q_{01}^{Pa} = \frac{V_{01}}{P_{01}^{La}}$$

or $P_{01}^{Pa} \times Q_{01}^{La} = V_{01}$ and $P_{01}^{La} \times Q_{01}^{Pa} = V_{01}$

ii). The Laspeyres and Paasche index forms are related so that the reciprocal of the forward Laspeyres index is the backward

Paasche index and the reciprocal of the forward Paasche index is the backward Laspeyres index.

i.e.

$$P_{10}^{Pa} = \frac{1}{P_{01}^{La}}$$

and

$$P_{10}^{La} = \frac{1}{P_{01}^{Pa}}$$

or $\qquad P_{10}^{Pa} \times P_{01}^{La} = 1$ and $P_{10}^{La} \times P_{01}^{Pa} = 1$

iii). From the first property we have

$$P_{01}^{Pa} \times Q_{01}^{La} = P_{01}^{La} \times Q_{01}^{Pa} = V_{01}$$

$$=> \frac{P_{01}^{Pa}}{P_{01}^{La}} = \frac{Q_{01}^{Pa}}{Q_{01}^{La}}$$

It follows that the ratio of the Paasche to the Laspeyres form is same for price and for quantity index numbers. Using this property it has been statistically proved that the Paasche price index is greater than the Laspeyres if prices and quantities tend to move in the same direction between years 0 and 1. The Laspeyres index is greater if prices and quantities tend to go in opposite directions.

iv). Laspeyres price index is based on the assumption that the quantities consumed in the base year and the current year are same. This assumption is not true in general. If the consumption of some of the commodities decreases in the current year due to rise in their prices or due to changes in the habits, tastes and customs of the people, then Laspeyres index gives relatively more weight-age to such commodities as it does not take into account falls in demand or changes in output. The numerator in the formula is relatively larger. Thus Laspeyres index number over-estimates the true value and is expected to have 'an upward bias'. Paasche, on the other hand tends to understate the rise in prices because it uses current weights.

In practice, neither all prices nor all quantities move in the same ratio and the relationship between the two systems depends on the correlation between the price and quantity movements, which is normally negative i.e. as one increases the other decreases. Therefore those goods which have risen in price more than others at a time when prices in general are rising-or the reverse-will tend to have current quantities relatively smaller than the corresponding base quantities and they will thus have less weight in the Paasche index. The Paasche index number under-estimates the true value and is expected to have 'a downward bias'.

The Laspeyres index is probably more convenient to use since it uses fixed weights. But it is found that with the passage of time these weights become out of date. The Paasche index uses the current weights. But it is sometimes difficult to obtain up-to-date information and in that case Laspeyres formula is more advantageous.

4. Other forms of index numbers

In search of perfect index number some compromises have been made and other formulae have been discovered.

i). Marshall-Edgeworth Price Index Number:

Here the weights attached to the commodities are taken as the arithmetic mean of the quantities in the base year and current year, i.e. $w = \frac{q_0+q_1}{2}$

$$P_{01}^{ME} = \frac{\sum p_1\left(\frac{q_0+q_1}{2}\right)}{\sum p_0\left(\frac{q_0+q_1}{2}\right)} \times 100$$

$$= \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$$

ii).Dorbisch-Bowley Price Index Number:

It is the arithmetic mean of Lapeyres and Paasche price index numbers.

$$P_{01}^{DB} = \frac{1}{2}[P_{01}^{La} + P_{01}^{Pa}]$$

$$= \frac{1}{2}\left[\frac{\sum p_1 q_0}{\sum p_0 q_0} + \frac{\sum p_1 q_1}{\sum p_0 q_1}\right] \times 100$$

<u>iii). Fisher Price Index Number:</u>

It is the geometric mean of the Laspeyres and Paasche price index numbers.

$$P_{01}^{F} = [P_{01}^{La} \times P_{01}^{Pa}]^{\frac{1}{2}}$$

$$= \left[\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}\right]^{\frac{1}{2}} \times 100$$

In general, the true value of the price index lies somewhere between the Laspeyres price index and the Paasche price index. Both Marshall-Edgeworth and Fisher index numbers lie between Laspeyres and Paasche indices. They have no bias in any known direction and provide a better estimate of the true price index.

Example 2: Using the data of example 1 compute

(a) Marshall-Edgeworth Price Index Number

(b) Dorbisch-Bowley Price Index Number

(c) Fisher Price Index Number

Solution:

	Base Year		*Current Year*	
Commodities	Price *per unit (Rs.)*	Expenditure *(Rs.)*	Price *per unit (Rs.)*	Expenditure *(Rs.)*

A	2	40	5	75
B	4	16	8	40
C	1	10	2	24
D	5	25	10	60

$$Expenditure = Price \times Quantity$$

$$=> Quantity = \frac{Expenditure}{Price}$$

Commodities	p_0	q_0	p_1	q_1	$p_0 q_0$	$p_1 q_1$	$p_0 q_1$	$p_1 q_0$
A	2	20	5	15	40	75	30	100
B	4	4	8	5	16	40	20	32
C	1	10	2	12	10	24	12	20
D	5	5	10	6	25	60	30	50

$$\sum p_0 q_0 = 91, \quad \sum p_1 q_1 = 199, \quad \sum p_0 q_1 = 92, \quad \sum p_1 q_0 = 202$$

(a) Marshall-Edgeworth Price Index Number:

$$P_{01}^{ME} = \frac{\sum p_1 \left(\frac{q_0 + q_1}{2}\right)}{\sum p_0 \left(\frac{q_0 + q_1}{2}\right)} \times 100$$

$$= \frac{\sum p_1 q_0 + \sum p_1 q_1}{\sum p_0 q_0 + \sum p_0 q_1} \times 100$$

$$= \frac{202 + 199}{91 + 92} \times 100$$

$$= 219.13$$

(b) Dorbisch-Bowley Price Index Number:

$$P_{01}^{DB} = \frac{1}{2}[P_{01}^{La} + P_{01}^{Pa}]$$

$$= \frac{1}{2}\left[\frac{\sum p_1 q_0}{\sum p_0 q_0} + \frac{\sum p_1 q_1}{\sum p_0 q_1}\right] \times 100$$

$$= \frac{1}{2}[221.98 + 216.3]$$

$$= 219.14$$

(c) Fisher Price Index Number:

$$P_{01}^{F} = [P_{01}^{La} \times P_{01}^{Pa}]^{\frac{1}{2}}$$

$$= \left[\frac{\sum p_1 q_0}{\sum p_0 q_0} \times \frac{\sum p_1 q_1}{\sum p_0 q_1}\right]^{\frac{1}{2}} \times 100$$

$$= [221.98 \times 216.3]^{\frac{1}{2}}$$

$$= 219.12$$

5. Fixed and Chain Base Index Numbers

The various formulae discussed for the construction of index numbers are based on the fixed base method. They reflect the relative changes in the level of a phenomenon in any period called the current period with respect to its value in some particular fixed year called the base year. Fixed base indices, though, simple to construct have some limitations. The draw-back of the fixed base method is that it requires the same set of commodities to be used in both the periods. The relative importance of the various commodities may change due to the changes in the consumption patterns of the society and as such weights need to be revised.

In view of these limitations, it was felt that the data for the two periods being compared should be as homogeneous as possible and this is best obtained by taking two adjacent periods. This requirement has given rise to chain base method where the relative changes in the level of phenomenon for any period are compared with that of the immediately preceding period and the process is continued till the comparison is made with the required base period.

i). Construction of chain base index number

$Change\ Base\ Index\ Number\ for\ any\ Year$

$$= \frac{Link\ Relative\ of\ Current\ Year \times Chain\ Base\ Index\ of\ the\ Preceding\ Year}{100}$$

where,

$Link\ Relative\ of\ any\ Year$
$= Price\ of\ that\ year\ as\ a\ percentage\ of\ its\ price\ in\ the\ preceding\ year$

$$=> Link\ Relative\ for\ i^{th} year = \frac{p_i}{p_{i-1}} \times 100, \quad i = 1,2, \dots \dots \dots, n$$

Example 3: From the following prices of three groups of commodities for the years 2001 to 2005, find the chain base index numbers chained to 2001.

Groups	2001	2002	2003	2004	2005
A	8	12	16	20	24
B	32	40	48	60	72
C	16	20	32	40	48

Solution:

Group	Link Relatives				
	2001	2002	2003	2004	2005
A B C	100 100 100	$\frac{12}{8} \times 100 = 150$ $\frac{40}{32} \times 100 = 125$ $\frac{20}{16} \times 100 = 125$	$\frac{16}{12} \times 100 = 133.33$ $\frac{48}{40} \times 100 = 120$ $\frac{32}{20} \times 100 = 160$	$\frac{20}{16} \times 100 = 125$ $\frac{60}{48} \times 100 = 125$ $\frac{40}{32} \times 100 = 125$	$\frac{24}{20} \times 100 = 120$ $\frac{72}{60} \times 100 = 120$ $\frac{48}{40} \times 100 = 120$
Total	300	400	413.33	375	360
(LR) Average	100	133.33	137.78	125	120
Chain Index	100	$\frac{100 \times 133.33}{100} = 133.33$	$\frac{137.78 \times 133.33}{100} = 183.70$	$\frac{125 \times 183.70}{100} = 229.63$	$\frac{120 \times 229.63}{100} = 275.56$

ii). Construction of fixed base index number from chain base index number:

Fixed base index (FBI) can be obtained from the chain base index (CBI) using the following formula:

$$\text{Current year FBI} = \frac{\textit{Current year CBI} \times \textit{Previous year FBI}}{100}$$

The FBI for the first period is same as the CBI for the first period.

Example: From the chain base index numbers given below, prepare fixed base index numbers:

Year:	**1990**	**1991**	**1992**	**1993**	**1994**
Index:	**90**	**120**	**130**	**105**	**150**

Using the above formula we obtain the FBI numbers as given in the following table:

Year	Chain Index Numbers	Fixed Base Index Numbers
1987	90	90
1988	120	$= \frac{90 \times 120}{100} = 108$
1989	130	$= \frac{108 \times 130}{100} = 140.40$
1990	105	$= \frac{140.40 \times 105}{100} = 147.42$
1991	150	$= \frac{147.42 \times 150}{100} = 221.13$

6. Summary

An index number is a measure of change in magnitude from one situation to another. The two situations may be two time periods, two regions of a country or two groups of individuals. For economic and business study index numbers may be broadly classified into three categories:

(1)Price Index Numbers

(2)Quantity Index Numbers

(3)Value Index Numbers

Different authors have given different formulae of constructing index numbers by adopting different procedures of assigning weights to the commodities. These index numbers are:

(1)Laspeyres Index Number

(2)Paasche Index Number

(3)Marshall-Edgeworth Index Number

(4)Dorbisch-Bowley Index Number

(5)Fisher Index Number

These fixed base indices have some limitations. The consumption pattern of the people change, there may be new innovations and the relative importance of the commodities may also change. In view of these limitations the chain base index number has been constructed which measures the changes in the level of a phenomenon for any period with that of the immediately preceding period and the process is continued till the comparison is made with the required base period.