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Production Function

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1. Introduction

Theory of production, explains the principles by which a business firm decides how much of each it will produce, and how much of each kind of labour, raw material, fixed capital good, etc., that it employs it will use. The theory involves some fundamental principles of economics such as the relationship between the prices of commodities and the prices (or wages or rents) of the productive factors that were used to produce them and also the relationships between the prices of commodities and productive factors.

The supply of a product depends upon its cost of production, which in turn depends upon (a) the physical relationship between inputs and output and (b) the prices of inputs. The theory of production relates to the physical law governing the production of goods. The act of production involves the transformation of inputs into outputs and the relation between inputs and output is known as the production function.

The production function of a firm can be examined by holding the quantities of some factors as fixed, while varying the quantum of other factors. This is done when the law of variable proportions is applied. The law of variable proportions is relevant for short run because in the short run some factors such as capital equipment, machines, land, etc remain fixed and factors such as labour and raw materials are increased to expand output.

Secondly, we study the input-output relation by varying all the inputs. This forms the subject matter for law of returns to scale, which is the long run production function.

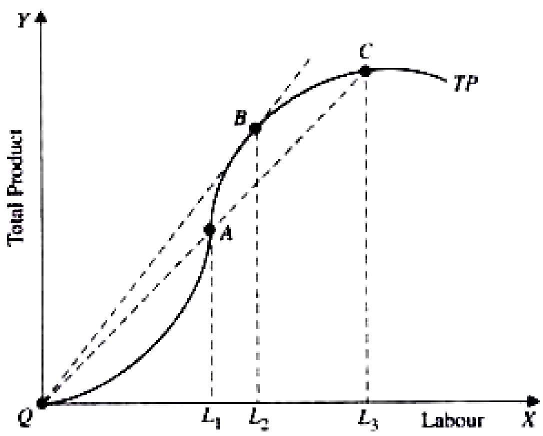
Total, Average and Marginal Physical Products

Three concepts related to physical production are: (1) Total Product (2) Average Product (3) Marginal Product.

Total Physical Product (TPP)

Total physical product (TPP) of a factor is the amount of total output produced by a given amount of the variable factor, and keeping supply of other factors like capital and capital as constant. As the amount of a factor increases, the total physical output increases.

In diagram 1 the total product physical curve is represented by TP curve. The diagram shows that in the initial stages, the TP curve rises at an increasing rate, that is, the slope of the TP curve is rising. However, after certain point, the TP curve increases at a diminishing.



Physical Product of Labour

Diagram 1

In diagram1 with employment of OL_1 quantum of labour the total output is L_1A units of the product. When the employment of labour increases to OL_2 and OL_3 then total output increases to L_2B and L_3C units of the product. The TP curve shows that further employment of labour will reduce the production of total physical product.

Average Physical Product

Average physical product of a factor is the per unit production of the factor that was used to produce the physical product. In other words average physical Product is measured by dividing the total physical product by total employment of factor input. If Q stands for total physical product, L for the number of a variable factor employed, then average product (AP) is given by:

$$AP_L = Q/L$$

From the TP curve in diagram 1, we can measure the average product of labour. When OL_1 units of labour are employed, production of total physical product is L_1A. Therefore, average product of labour equals L_1A/OL_1, which would be equal to the slope of the ray OA. Similarly, when OL_2 units of labour are employed, total product (TP) is L_2B and average product will be L_2A/OL_2, which is the slope of ray OB. And by employing OL_3 units of labour, the average product will be measured by the slope of the ray OC.

Marginal Physical Product

Marginal physical product of a factor is an addition to the total production by the employment of an extra unit of a factor. Mathematically, if employment of labour increases by ΔL units due to which total output increases by ΔQ units, then marginal physical product of labour MP_L is calculated by

$$MP_L = \Delta Q/\Delta L$$

At any given level of labour employment, the marginal product of labour can be obtained by measuring the slope of the total product. For example, in Diagram 1 when OL_1 units of labour are employed, the marginal physical product of labour is given by the slope of the tangent drawn at point A to the total product curve TP. Similarly, when OL_2 units of labour are employed, the marginal physical product of labour can be obtained by measuring the slope of the tangent drawn to the total product curve TP at point B which corresponds to OL_2 level of labour employment.

The marginal physical product of a factor will change at different levels of employment of the factor. It has been found that marginal product of a factor rises in the beginning and then ultimately falls and become negative as more and more of it is used for production, other factors remaining the same.

Law of Variable Proportions

Law of variable proportions examines the production function with one factor variable, keeping the quantities of other factors fixed. Thus it refers to the input-output relation. Since under this law we study the effects on output by a change in factor proportions, this is also known as the law of variable proportions. The law of variable proportions is the new name given to the 'Law of Diminishing Returns' of the classical economics.

Assumptions of the Law of Variable Proportions

The law of variable proportions or diminishing returns, as stated above, is subject to the following conditions:

1. Technology is given and do not change. With improved technology, the marginal and average products will rise.
2. LVP assumes given and fixed quantity of some inputs. This law does not apply when all factors are variable.
3. LVP is based on the possibility of varying the proportions the proportions of factor inputs. The law does not apply to those cases where the factors must be used in fixed proportions to yield a product.

Three Stages of Law of Variable Proportions

The behaviour of output when the varying quantity of one factor is combined with a fixed quantity of the other can be divided into three distinct stages. These three stages are explained with the help of diagram 2, which illustrate the production function with one factor variable.

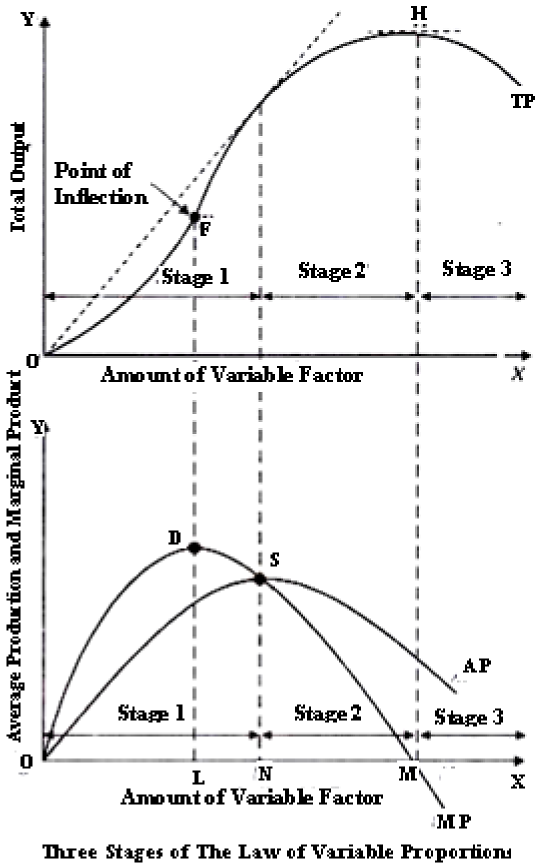


Diagram 2

In diagram 2, the quantity of the variable factor is measured on X-axis while the total, average and marginal products are measured on Y-axis. How the total product, average product and marginal product of a variable factor change due to the increase in its quantity? This is seen on the diagram by increasing the quantity of one factor when the quantity of the other factors is constant.

In the top panel of diagram 2, the total product curve TP of the variable continuously increases upto a point and thereafter it starts declining. In the bottom panel, average product curve AP and marginal product curve MP of labour also rise and then decline. The marginal product curve starts declining earlier than the average product curve.

The behaviour of these total, average and marginal products of the variable factor as a result of the increase in its amount is generally divided into three stages, which are explained below:

Stage1: Increasing Returns

In this stage, total product curve TP increases at an increasing rate up to point F in diagram 2. From the origin to point F, the slope of the TP is increasing which states that total physical product increases at an increasing rate. Thus the TP curve is concave upward up to point F, which means that the marginal product MP of the variable factor is rising.

From point F onwards during stage 1, the total product curve continue rising but its slope is declining which indicate that from point F onwards the total product increases at a diminishing rate. Thus the TP curve concave downward as the marginal physical product falls but remains positive.

The point F where the total product stops increasing at an increasing rate and starts increasing at the diminishing rate is called the point of inflection. Vertically corresponding to this point of inflection, the marginal physical product is maximum, after which it starts diminishing.

The marginal physical product of the variable factor starts diminishing beyond OL units of its input. That is, law of diminishing returns starts operating in stage1 from point D on the MP curve or from OL amount of variable factor input.

The first stage ends where the average product curve AP reaches its highest point, which is point S on AP curve where ON quantum of the variable factor input is used. During stage 1, when marginal product of the variable factor falls (MP negative but still exceeds its average product), the average product curve AP continues to rise.

Stage 2: Diminishing Returns

In stage 2, although TP curve continues to increase, but the increase is at a diminishing rate until it reaches its maximum point H where the second stage ends. In this stage both the marginal product and the average product of the variable factor are diminishing but both remain positive. At the end of second stage, which reaches at point M on MP curve, the marginal physical product of the variable factor is zero point M, which corresponds to the highest point H on the TP curve.

Stage 3: Negative Returns

In stage 3, with the increase in variable factor, the total physical product declines which cause a downward slopes in the TP curve. In this stage the marginal physical product of the variable factor is negative and the MP curve moves below the X-axis. This stage is called the stage of negative returns, since the marginal product of the variable factor is negative during this stage.

Causes of Increasing Marginal Returns to a Factor

In the initial stage of production, the input of fixed factor is relatively high because technology or machinery demand some minimum employment of fixed factor irrespective of the level of output. Therefore, when more units of the variable factor are employed to work with an indivisible fixed factor, the output will increases due to better utilization of the fixed factor. As a result there are increasing marginal returns to the variable factor. The second reason for increasing returns to the variable factor is that as more units of the variable factor are employed the efficiency of the variable factor itself increases due division of labour as per their skill.

Causes for Diminishing Marginal Returns to Factors

Diminishing Marginal Returns to a Factor occur due to three reasons. They are:

1. Scarcity of Fixed Factor: When the fixed factor is optimally used it becomes scarce in relation to the increase in variable factor. Scarcity of fixed factor cause a decline in the marginal and average products of the variable factor.

2. Indivisibility of Fixed Factor: When the indivisible fixed factor is fully utilized, the use of factor inputs at this level is called "best or optimum proportion" with the variable factor. Once the optimum proportion is disturbed by further increases in the variable factor, returns to a variable factor (i.e., marginal product and average product) will diminish. If the so called fixed factors were perfectly divisible, then the question of varying factor proportions would not have arisen.

3. Imperfect Substitution of Factors: the factors of production can be substituted for one another within certain limits. Thus, the elasticity of substitution between factor is not infinite. Thus

diminishing returns operate because the elasticity of substitution between factors is not infinite.

Production Functions with Two Variable Factors

The functional relation between physical inputs and physical output of a firm is known as the production function. For the analysis of production function with two variable factors, the concepts of iso-quants or iso-product curves are used which are similar to indifference curves of the theory of demand. If there are two inputs, capital (K) and labour (L), then the production function is written as Q = f (L, K). This function defines the output (Q) that can be produced with available quantities of capital and labour inputs.

Isoquants

An isoquant represents all those factor combinations which are capable of producing the same level of output. The concept of isoquant is explained with the following table.

Table1: Factor Combinations for Given Level of Output

Factor Combinations	Labour	Capital
A	1	12
B	2	8
C	3	5
D	4	3
E	5	2

Suppose two factors, labour and capital, are employed to produce a product. Each of the factor combinations A. B, C, D and E produces the same level of output, say 100 units. To start with, factor combination A consisting of 1 unit of labour and 12 units of capital produces the given 100 units of output. Likewise combination E consisting of 5 units of labour and 2 units of capital are capable of producing the same amount of output, i.e., 100 units.

All these combinations when plotted in diagram 3 and when joined with one another will generate an isoquant which states that every combination of inputs represented on it can produce same level of output i.e. 100 units.

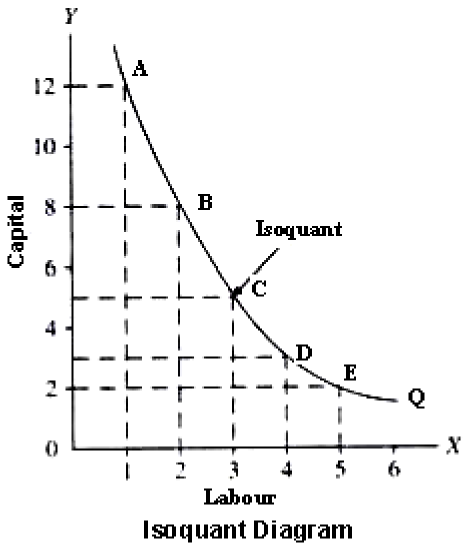


Diagram 3

Isoquants can be labeled in physical units of output without any difficulty. Production of a good it is measured in absolute measurement of physical units. Since each isoquant represents a specified level of production, it is possible to say by how much production one isoquant represents.

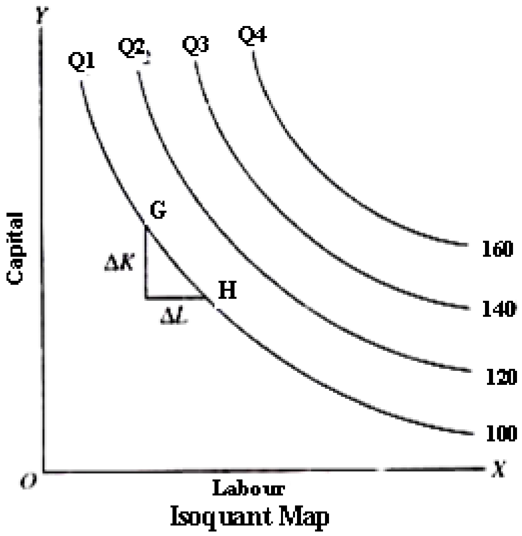


Diagram 4

Diagram 4 shows an isoquant-map or equal- product map with a set of four isoquants, which represent 100 units, 120 units, 140 units and 160 units of output respectively. Then, from this set of isoquants it is very easy to judge by how much production level, one isoquant curve is greater or less than on another.

General Properties of Isoquants:

The isoquants normally possess properties which are similar to those assumed for indifference curves. Moreover, the properties of isoquants can be proved in the same manner as in the case of indifference curves. The following are the important properties of isoquants:

1.Isoquants slope downward because when the quantity of a factor, say labour, is increased, the quantity of other capital i.e., capital must be reduced so as to keep output constant on a given isoquant. This downward-sloping property of isoquants follows from a valid assumption that the marginal physical products of factors are positive.

2. No two isoquants can intersect each other. If the two isoquants, intersect each other, there will then be a common

factor combination corresponding to the point of intersection. How can the same factor combination produce two different levels of output when the technique of production remaining unchanged?

3. Isoquants are convex to the origin indicates. The convexity of equal product curves is due to the diminishing marginal rate of technical substitution of one factor for the other.

Returns to Scale

The study of changes in output due to the changes in the scale forms the subject matter of 'returns to scale'. The returns to scale may be constant, increasing or decreasing. If all the factors (i.e., scale) in a given proportion and the output are increases in the same proportion, returns to scale are said to be constant. Similarly if a given percentage increase in all factors leads to more than proportionate increase in output, the returns to scale are said to be increasing. Likewise if the increase in all factors leads to a less than proportionate increase in output, returns to scale are decreasing.

Constant Returns to Scale

The constant returns to scale means that with a given percentage increase in the scale or the amounts of all factors leads to the same percentage increase in output, that is, doubling of all inputs doubles the output.

Empirical evidence suggests that production function for the economy as a whole is not too far from being homogeneous of the first degree. Empirical evidence also suggests that in the production function for an individual firm, there is a long phase of constant returns to scale.

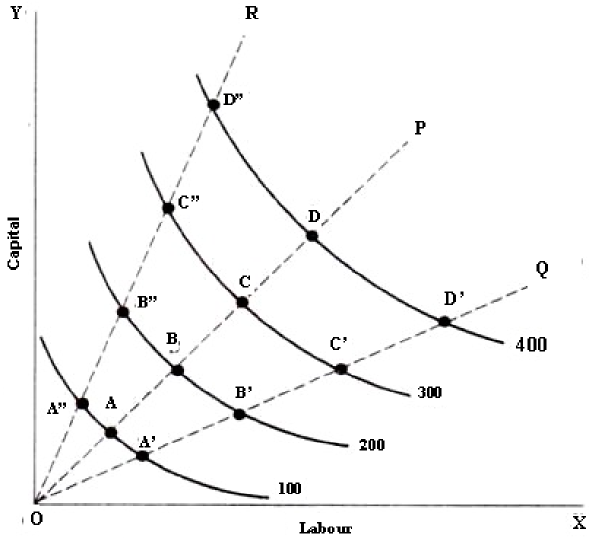


Diagram 5

Diagram 5 illustrates the constant returns to scale with the help of equal product curves i.e. isoquants. The diagram depicts an isoquant map. In order to understand whether or not returns to scale are constant, some straight lines through the origin are drawn.

These straight lines passing through the origin indicate the increase in scale as we move upward. It can be seen from the diagram that successive isoquants are equidistant from each other along each straight line drawn from the origin. Thus along the line OP, AB = BC = CD. Likewise along line OQ and line OR, the isoquants are at equidistance from each other. The distance between the successive equal product curves being the same along any straight line through the origin means that if both labour and capital are increased in a given proportion, output expands by the same proportion. Therefore, diagram 5 displays constant returns to scale.

Increasing Returns to Scale

Increasing returns to scale means that output increases in a greater proportion than the increase in inputs. If, for instance, all inputs are increased by 25%, and output increases by 40% then the increasing returns to scale will be prevailing.

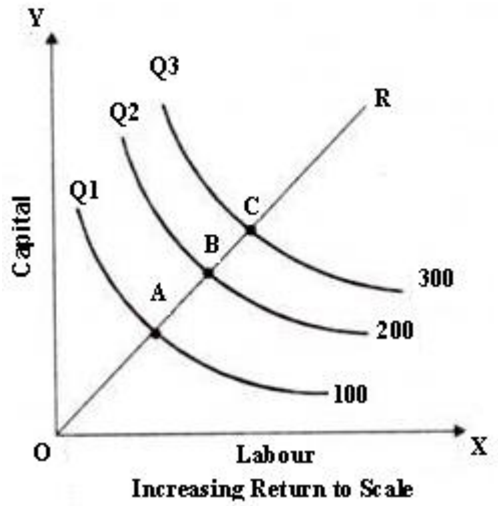


Diagram 6

When increasing returns to scale occur, the successive isoquants will lie at decreasingly smaller distances along a straight line ray OR through the origin. In diagram 6 the isoquants Q_1, Q_2, Q_3 are drawn which successively represent 100, 200 and 300 units of output. The diagram shows that the distances between the successive isoquants decrease as output expands by increasing the scale. Thus, increasing returns to scale occur because distance AB < OA and BC < AB which means that equal increases in output are obtained by smaller and smaller increments in inputs.

Decreasing Returns to Scale

When output increases less than proportion to the increase in all inputs, decreasing returns to scale prevail. When a firm goes on expanding by increasing all his inputs, eventually diminishing returns to scale will take place.

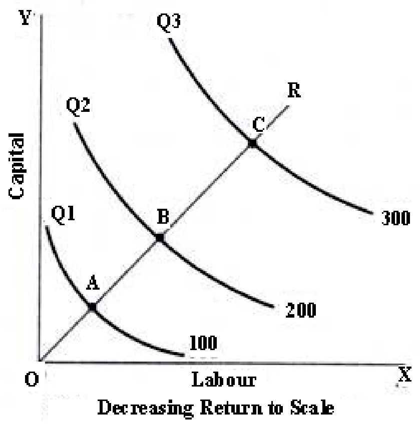


Diagram 7

When successive isoquants lie at progressively larger and larger distance on a ray through the origin, returns to scale will be decreasing. In diagram 7 decreasing returns to scale occur since AB > OA, and BC > AB. It means that successively more and more of inputs (labour and capital) are required to obtain equal increments in output.

Summary

The supply of a product depends upon its cost of production, which in turn depends upon the physical relationship between inputs and output and the prices of inputs. For the analysis of production function with two variable factors, the concepts of iso-quants or iso-product curves are used which are similar to indifference curves of the theory of demand. An isoquant represents all those factor combinations which are capable of producing the same level of output. Isoquants can be labeled in physical units of output without any difficulty as the production of a good is measured in absolute measurement of physical units. Since each isoquant represents a specified level of production, it is possible to say by how much production one isoquant represents. The isoquants normally possess properties which are similar to those assumed for indifference curves. Moreover, the properties of isoquants can be proved in the same manner as in the case of indifference curves.