**PHYSICAL EDUCATION**

Subject : Physical Education
Semester : $4^{th}$
Paper No. and Title : (401) Test, Measurement and Evaluation in Physical Education

## Unit-3: Measures of Central Tendency

### 3.0 Objective:

In Statistical analysis, generally we seek to develop brief summary figures, which describe a set of data adequately. Measures of central tendency are most widely used summary figures in Statistics. This unit considers these measures along with illustrations. After reading this unit, we will be able to:

- define measures of central tendency
- understand the importance of measures of central tendency
- calculate mean, median and mode for grouped and ungrouped data

### 3.1 Meaning and Importance Central Tendency:

Presentation of data by the frequency table is not enough to provide much knowledge about the nature of the data. Due to which, we need some other techniques by which we can acquire more information from the data set. One of such techniques is central tendency. When we observe carefully a set of data, then we find a tendency of the individual value to cluster around the central value. This type of tendency of the individual value is called central tendency and when we measure this tendency by a particular technique, this value is called a measure of central tendency. It is a statistical measure to determine a single score that define the centre of a distribution. The main focus of central tendency is to give a single value which is most representative of the data set.

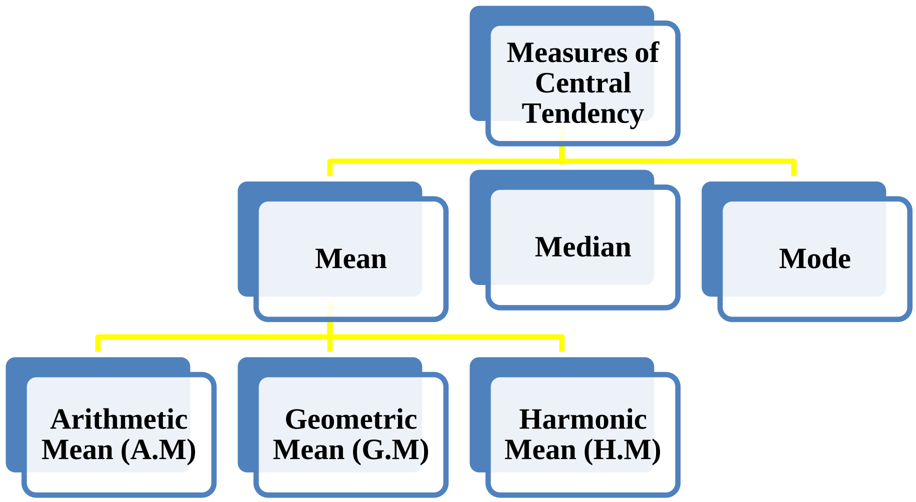


**Fig. 3.1: Measures of Central Tendency**

There are three types of measures of central tendency- mean, median and mode. The mean can be further divided into three measures- arithmetic mean (A.M.), geometric mean (G.M.) and harmonic mean (H.M.). These measures reduce the large gathering of observations to a single figure for which central tendency is an essential and important summery measure in statistics. An average makes it handy to express the data in the way that the significant characteristics of the data come to the light. It is difficult to generalize anything from the height of people of India whereas if we say the average height of the people is 168 cm than one can draw some conclusion by this result. Again average gives us a common score to compare one set of data to other. For example, we can calculate the average score of three cricket players in a series of five matches and by comparing these three averages, we can comment on the performance of each player. In some cases of statistics, where it is not relevant to collect the whole data of the population, the investigator just collects the sample data from that population. In this case, the sample mean gives a good idea about the mean of the population.

**3.2 Mean:**

Mean is the most widely used measure of central tendency. Mean which is also known as average, generally refers to the arithmetic mean of a data set. The individual observations are sum-up as a whole and the result, thus obtained is divided by the total number of observations, which give us a single value- mean. In calculation of mean, each value or observations have equal impact on the result. Mathematically, the calculations of mean for grouped and ungrouped data are as follows:

**Ungrouped Data**

If the variable X takes the values $x_1, x_2, x_3, ..., x_n$ , then the mean of X is given by,

$$\bar{x} = \frac{x_1+x_2+x_3 \ldots +x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2.1$$

Where, n = total number of observations

**Grouped Data**

If the variable X takes the values $x_1, x_2, x_3, ..., x_n$ (or mid values of the class intervals) with corresponding frequencies $f_1, f_2, f_3, ..., f_n$ , then the mean of X is given by,

$$\bar{x} = \frac{f_1x_1+f_2x_2+f_3x_3 \ldots +f_nx_n}{f_1+f_2+f_3 \ldots +f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{N} \ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2.2$$

Where, N = $\sum_{i=1}^{n} f_i$ = total frequency

**Example 3.2.1**

The marks obtained by 30 students in a class test, out of 50 marks, according to their roll numbers are:*41,25,5,33,12,21,19,39,19,21,12,1,19,12,19,17,12,17,17,41,41,19,41,33,12,21,33,5,1,21*. Calculate the average marks.

**Solution:**

| | | | | | |
|---|---|---|---|---|---|
| Roll No. | 1 | 2 | 3 | 4 | 5 |
| Marks ($x_i$) | 41 | 25 | 5 | 33 | 12 |
| Roll No. | 6 | 7 | 8 | 9 | 10 |
| Marks ($x_i$) | 21 | 19 | 39 | 19 | 21 |
| Roll No. | 11 | 12 | 13 | 14 | 15 |
| Marks ($x_i$) | 12 | 1 | 19 | 12 | 19 |
| Roll No. | 16 | 17 | 18 | 19 | 20 |
| Marks ($x_i$) | 17 | 12 | 17 | 17 | 41 |
| Roll No. | 21 | 22 | 23 | 24 | 25 |
| Marks ($x_i$) | 41 | 19 | 41 | 33 | 12 |
| Roll No. | 26 | 27 | 28 | 29 | 30 |
| Marks ($x_i$) | 21 | 33 | 5 | 1 | 21 |

Here,

Total no. Of observations, n = 30

Grand total of marks = 629

Therefore,

$$\text{Mean, } \bar{x} = \frac{\sum_{i=1}^{n} x_i}{N}$$

$$= \frac{629}{30}$$

$$= 20.97$$

Hence, the average marks of the students is 20.97

**Example 3.2.2**

Calculate the average of the following:

| Marks obtained | No. Of Students |
|---|---|
| 0-10 | 4 |
| 11-20 | 13 |
| 21-30 | 5 |

| | |
|---|---|
| 31-40 | 4 |
| 41-50 | 4 |

**Solution:**

| Marks obtained | Mid Value ($x_i$) | No. Of Students ($f_i$) | $f_i x_i$ |
|---|---|---|---|
| 0-10 | 5 | 4 | 20 |
| 11-20 | 15 | 13 | 195 |
| 21-30 | 25 | 5 | 125 |
| 31-40 | 35 | 4 | 140 |
| 41-50 | 45 | 4 | 180 |
| Total | - | 30 | 660 |

Hence,

$N = \sum_{i=1}^{n} f_i = 30$

$\sum_{i=1}^{n} f_i x_i = 660$

Therefore,

Mean, $(\bar{x}) = \frac{\sum_{i=1}^{n} f_i x_i}{N} = \frac{660}{30} = 22$

Thus, the average mark of the students is 22.

Like arithmetic mean or simply mean as discussed above, harmonic mean and geometric mean are two other types of mean which have limited application compared to arithmetic mean. Harmonic mean generally use in calculation of speed in physics on the other hand the uses of geometric mean can be seen in the calculation of index numbers in business and economics.

**3.3 Median:**

Median of a data set is that value, which divides the data set into two equal parts after arranging it either in ascending order or in descending order It is the value such that the number of observations above it is equal to the number of observations below it. The median is thus a positional average. Mathematically, the calculations of median are as follows:

## **Ungrouped data**

In this case, first we arrange the data in order (ascending or descending)

Let, n= no. of observations is an odd number, then

Median = $\frac{n+1}{2}^{th}$ observation

In case of even number of observations,

Median= mean of $(\frac{n}{2})^{th}$ observation and $(\frac{n}{2}+1)^{th}$ observation, i.e., arithmetic mean of the two middle terms.

## **Grouped data**

In the case of grouped frequency table, the class corresponding to the cumulative frequency (c.f.) just greater than or equal to $\frac{N}{2}$ , is called the median class and the value of median is obtained by:

$$\text{Median} = l + \frac{h}{f}\left(\frac{N}{2} - c\right)$$

Where, $l$ is the lower limit of the median class

$f$ is the frequency of the median class

$h$ is the length of the median class

c is the c.f. of the class preceding the median class, and $N = \sum_i f_i$

**Example 3.3.1**

Find the Median of

(I) 10,6,17,12,13,8,21,9,2,14,4
(II) 10,6,17,12,13,8,21,9,2,14,4,25

Solution:

(I) Here n=11, which is odd

After arranging in ascending order we get,

2,4,6,8,9,10,12,13,14,17,21

Therefore,

Median = value of $(n+1)/2^{th}$ item

= value of (11+1)/2 item

= value of $6^{th}$ item

= 10

(ii) Here n=12, which is even

After arranging in ascending order we get,

2,4,6,8,9,10,12,13,14,17,21,25

Therefore,

$$\text{Median} = \frac{\text{value of } \left(\frac{n}{2}\right)\text{th item} + \text{value of } \left(\frac{n}{2}+1\right)\text{th item}}{2}$$

= (value of $6^{th}$ item + value of $7^{th}$ item)/2

= (10 +12)/2

= 22/2

=11

**Example 3.3.2**

The marks obtained by 30 students in a class test, out of 50 marks, according to their roll numbers are:*41,25,5,33,12,21,19,39,19,21,12,1,19,12,19,17,12,17,17,41,41,19,41,33,12,21,33,5,1,21*. Calculate the median of the marks.

**Solution:**

| Marks($X_i$) | Frequencies($f_i$) | Cumulative |
|---|---|---|

| | | Frequencies(c.f.) |
|---|---|---|
| 1 | 3 | 3 |
| 5 | 2 | 5 |
| 12 | 5 | 10 |
| 17 | 3 | 13 |
| 19 | 5 | 18 |
| 21 | 4 | 22 |
| 33 | 3 | 25 |
| 39 | 1 | 26 |

Here N=30, $\frac{N}{2}$ =15

The cumulative frequency (c.f.) just greater than $\frac{N}{2}$ is 18 and the value of x corresponding to 18 is 19. Therefore median is 19.

**Example 3.3.3**

Calculate the Median of the following:

| Marks obtained | No. Of Students |
|---|---|
| 0-10 | 4 |
| 11-20 | 13 |
| 21-30 | 5 |
| 31-40 | 4 |
| 41-50 | 4 |

**Solution:**

| Marks obtained | No. Of Students | c.f. |
|---|---|---|
| 0-10 | 4 | 4 |
| 11-20 | 13 | 17 |
| 21-30 | 5 | 22 |

| | | |
|---|---|---|
| 31-40 | 4 | 26 |
| 41-50 | 4 | 30 |
| Total | N=30 | |

Here N=30, $\frac{N}{2}$ =15. Cumulative frequency just greater than 15 is 17 and the corresponding class is 11-20. Thus median class is 11-20.

Hence,

l=11

h= 10

f= 13

c=4

Therefore, Median=11+$\frac{10}{13}$(15-4)

=19.46

**3.4 Mode:**

Mode is the value which occurs most frequently in a set of observations i.e., the value having maximum frequency. In other words, mode is the value of the variable which is predominant in the series.

**Ungrouped data**

For ungrouped data mode is the value of x corresponding to the maximum frequency.

**Grouped data**

In case of continuous frequency distribution, mode id given by the formula:

$$\text{Mode } = l + \frac{h(f_1 - f_0)}{(f_1 - f_0) - (f_2 - f_1)} = l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2},$$

Where,

$l$ is the lower limit of the modal class

$h$ is the magnitude of the modal class

$f_1$ is the frequency of the modal class

$f_0$ is the frequency of the class preceding the modal class

$f_2$ is the frequency of the class succeeding the modal class

The class having maximum frequency is the model class.

**Example 3.4.1:** The marks obtained by 30 students in a class test, out of 50 marks, according to their roll numbers are:
*41,25,5,33,21,21,19,39,19,21,12,1,19,12,21,17,12,17,17,41,41,19,41,33,12,21,33,5,1,21*.

Calculate the mode value.

**Solution:**

| Marks(Xi) | Frequencies(fi) |
|---|---|
| 1 | 3 |
| 5 | 2 |
| 12 | 4 |
| 17 | 3 |
| 19 | 4 |
| 21 | 6 |
| 33 | 3 |
| 39 | 1 |
| 41 | 4 |
| Total | N=30 |

Here the value corresponding to maximum frequency 6 is 21. Hence mode is 21.

**Example 3.4.2**

Calculate the Mode of the following:

| Marks obtained | No. Of Students |
|---|---|

| | |
|---|---|
| 0-10 | 4 |
| 11-20 | 13 |
| 21-30 | 5 |
| 31-40 | 4 |
| 41-50 | 4 |

**Solution:**

| Marks obtained | No. Of Students($f_i$) |
|---|---|
| 0-10 | 4 |
| 11-20 | 13 |
| 21-30 | 5 |
| 31-40 | 4 |
| 41-50 | 4 |
| Total | N=30 |

Here maximum frequency is 13. Thus the class 11-20 is the modal class.

Hence,

$l = 11$

$h = 10$

$f_1 = 13$

$f_0 = 4$

$f_2 = 5$

Thus, Mode $= 11 + \frac{10(13-4)}{2(13)-4-5} = 11 + \frac{90}{17} = 16.29$