Design of Structures II

Lecture 9

Welcome to UGC lecture series in B.Architecture. Here, we are going to have a look at, Design of Structures II. In the previous lecture we have seen the analysis of a singly reinforced flanged beam. When the neutral axis lies within the flange i.e x is lesser than or equal to df and when the neutral axis lies outside the flange, what will be the moment of resistance of the respective sessions we have seen. Using this formula, we are going to do some example problems to find out the moment of resistance of a singly reinforced rectangular beam when the neutral axis lies within the flange and the neutral axis lies outside the flange.

The first problem; Find the moment of resistance of the T beam i.e so here, they have given a T beam. The size of the T beam i.e effective depth of T beam 400mm i.e a singly reinforced T beam, d = 400mm and the flange width is 1200mm and depth of flange is 100mm, df = 100mm, width i.e bw = 200mm and tensile steel consists of 4 numbers of 18 diameter i.e 4 x 18mm dia bars i.e Ast. Since it is a singly reinforced rectangular section, the reinforcement is provided at the tension zone only. We need to analysing this one. Here we are having two cases; one is neutral axis lies within the flange and neutral axis lies outside the flange. They have asked us to find out what is the moment of resistance of the beam. We do not know whether this beam occurs in the first case or in the second case? For that, we first begin with the assumption that the neutral axis lies within the flange. Assume neutral axis lies within the flange, let us start with the first case i.e x </= Df. First to find out xc;

This is the depth of neutral axis for the balanced section in order to find out whether our section is under reinforced or over reinforced section. So here, let us start with the assumption that the neutral axis lies within the flange. So here,

σ cbc = 7 N/mm ^{2}

σst = 190 N/mm ^{2}

m = 13.33

xc = 131.74 mm

After using the calculator to calculate, we will get the value for xc. Now, the neutral axis lies within the flange we have assumed. To find x:

This is the expression used to find out the actual depth of neutral axis for the first case, neutral axis lies within the flange.

After solving this, I get;

x = 84.48 mm

So here, x = 84.48mm. We need to first immediately compare the depth of neutral axis with the df. Now, we need to confirm whether the neutral axis lies within the flange or outside the flange. Here, x is less than 100mm i.e Df. Here, x is less than Df, assumption is correct i.e neutral axis lies within the flange and also, x is less than xc, it is under reinforced section. Now, if it is under reinforced section, in the case of $x < xc$, it is under reinforced. In the under reinforced section, the steel is stressed to its maximum permissible value.

$\sigma_{st} = 190 \text{ N/mm}^2$

Actual σ_{cbc}:

The actual σ_{cbc} obtained from this formula i.e σ_{cbc} (ac) = 3.82 N/mm ^{2}, this should be less than the permissible σ_{cbc} - 7 N/mm ^{2}. Now, the moment of resistance.

We can use this expression also to find out the moment of resistance of the under reinforced section. Since here, we have also found actual σ_{cbc} i.e equal to;

= 72.01 kNm

We can also use the expression;

$= T \times Z = 0.87 f_y A_{st} (d - x/3)$

= 72.01 kNm

Since, this is the beam which designed as a rectangular, we can use this expression to find out the moment of resistance once you find out the actual σ_{cbc} which must be less than the permissible σ_{cbc}. Next problem, a T beam has a permissible flange width of;

bf = 1500mm

Df = 100mm

d = 400mm, effective depth for the thickness

bw = 200mm

and the beam is reinforced on the tension side with 2190 Ast.

Ast = 2190 mm ^{2}

$\sigma_{cbc} = 5$

$\sigma_{st} = 140 \text{ N/mm}^2$

Now, you need to find out the moment of resistance of the section. First, we don't know whether the neutral axis lies within the flange or outside the flange. So first, let us start with;

X </= Df

Assume Neutral axis lies within the flange. To find out xc;

xc = 159.66 mm

To find out x;

x = 121.33 mm

This is greater than Df, the assumption is incorrect. Assumption is wrong, so here the neutral axis lies outside the flange. Case 2 - x > Df, we need to find out new x for the corresponding case i.e

$b_f D_f (x - D_f/2) = m A_{st} (d - x)$

So here, when the neutral axis lies outside the flange, the beam has to be analyzed as a T section i.e

1500 x 100 (x - 100/2) = 18.66 x 2190 (400 - x)

x = 124.84 mm > Df

So here x is greater than Df and also x is, we need to compare x with xc. x is 159.66 but here x is 124.94, x < xc, it is under reinforced section. In the case of under reinforced section, the nature of failure is a ductile failure, tension failure, we know σ st is known since the steel is stressed to its maximum permissible value, σ st is known which is equal to 140 N/mm ^{2}. So in the case of under reinforced section, the concrete is stressed below the permissible value. We need to find out actual σcbc i.e

For the depth of x, what will be the actual σ cbc?

σ cbc(ac) = 3.41 N/mm ^{2} which must be less than the permissible σ cbc i.e 5 N/mm ^{2}. Now, we need to find out the moment of resistance of the respective section, the moment of resistance is;

Mr = bf.Df i.e area of the portion

Now, we need to first find out what is σ cbc'. σcbc' is at the bottom of the flange i.e $\frac{\sigma cbc}{x} = \frac{\sigma cbc}{x}$ - Df. So here, this is the stress diagram, this is the flange portion Df, this is actual σ cbc, I am finding σ cbc' at the bottom of the flange. This is actual σ cbc'. This is at a depth of x from the neutral axis and this is at a depth of (x - Df), at a depth of (x - Df) compressive stress at the bottom of the flange is σ cbc'. At a depth of x from the neutral axis, the actual is here at the top it is σ cbc (ac).

σ cbc' = 0.68 N/mm ^{2}

What is ȳ?

ȳ = σ cbc +

Here it is σcbc, this one is taken as cg of this trapezoidal portion, if you take this trapezoidal portion, this is σ cbc actual and this is σ cbc' and this is one is Df and this is the cg of the portion ȳ.

This is CG of the trapezium. So ȳ here it is equal to 38.87mm . Now next one is Moment of resistance of the section.

So here by substituting all the things, we get;

= 110.776 kNm

This is the way of calculating the moment of resistance for the second case i.e x > Df. We got this expression while finding out the moment of resistance. So, since x > Df, it falls outside the flange. So it has to be analyzed as a T beam and it also has to be analyzed as a T section. Now, that is all about the singly reinforced flanged portion. Next, we start analyzing doubly reinforced T and L sections. Now, how to find out the moment of resistance for the doubly reinforced T and L section. Doubly reinforced means the section that has been reinforced both at a compression as well as at the tension zone is called the doubly reinforced means the section which is reinforced both at the compression as well as at the tension zone is called doubly reinforced section. So, here how to analyze the doubly reinforced T or C section. Here also there are three sections; one is balanced section, under reinforced section and over reinforced section. Reinforced concrete section, the design on the basic assumption that the both the materials are stressed to their maximum permissible value one and the same time. The section will be called as Balanced section. So here, the depth of neutral axis will be equal to the critical depth of neutral axis. In the case of under reinforced section, we are using less steel. If you less steel than what is required for the balanced section, the section will be called as under reinforced section. So here, in this section, steel is stressed to its maximum permissible value while the concrete is stressed below the permissible value. This section will initially fail due to over stress in steel. So the type of failure here is, the tensile failure or ductile failure. So in the case of under reinforced section, it is an unbalanced section. Neutral axis is going to be situated at the centre of gravity of the section. Here, the neutral axis moves upwards, so x < xc. So in the case of over reinforced, here we are using more steel. If more steel than what is required for the balanced section is used, that section will be over reinforced section. In such conditions, the section will fail initially due to over stress in concrete. So the type of failure that occurs here is, brittle failure or a sudden failure. This is also an unbalanced section, neutral axis is going to be situated at the centre of gravity of the section. So here, the neutral axis moves downwards, so x > xc. If x > xc, it is an over reinforced section. If it is less than xc, it is under reinforced section. If both are same, it is called as a balanced section. Now, how to find out the moment of resistance for the doubly reinforced section?

Here also two cases; one is x </= Df, another one is x > Df. So here, we need to add the moment of resistance due to the compressive steel. One is neutral axis lies within the flange or just as the part above the flange i.e x </= Df, we are going to find out x and what will be the moment of resistance of the beam. Now, case 1;

x </= Df, now to find out the moment of resistance of the doubly reinforced section. Since it is a doubly reinforced section, the beam is reinforced with the reinforcement both at the top as well as at the bottom. This is bf and this is df and here, this reinforcement is called as; area of compression reinforcement which is placed at d' from the topmost extreme fibre and this is the tension reinforcement which is called as Ast which is placed at d from the topmost extreme fibre and this one is bw without the section. Now, we need to find out what will be the moment of resistance of the section. Now I am taking this as the neutral axis for the balanced section. So here, the neutral axis normally occurs here. In the case of balanced section both the materials are stressed to their maximum permissible stresses one at the same time. At the top it is σ cbc i.e the stress in concrete surrounding concrete it is σ cbc, this is the equivalent stress diagram

for balanced section. Now here, the neutral axis lies within the flange. Now, here I am taking the equivalent stress diagram for unbalanced section. Here I am taking this as x and this as (d-x). Now here, first to find xc. xc using this stress diagram by using the similar triangle property at a depth of xc from the neutral axis. If you take this as xc, the stress as σ cbc, at a depth of (d-xc)

This is the expression which is used to find out x. Now to find out x in the case of the unbalanced section. The neutral axis is situated at the centre of gravity of the section, to find out x we are going to equate. By equating moment of area on compression side to the moment of area on or moment of equivalent concrete area on Tensile side. Now, this is the compression zone, it consists of both concrete as well as compression steel. Now we need to find out, first I am taking the whole concrete. The whole concrete means, the moment of area is;

Area x distance of C.G

What is the area of this compression zone?

bf. x . x/2

The distance of C.G from the neutral axis is x/2, it occurs at the centre. It is a whole concrete area. So here, since the steel is placed at the compression zone, we need to deduct the moment of area of compression steel, the distance of C.G of this compression steel here it is d', this one is (x - d'), the distance of C.G for the steel from the neutral axis i.e;

bf. x. x/2 - Asc(x - d') +

So we have deducted the moment of area of the compression steel. Now we need to add the equivalent concrete area of the compression steel i.e

bf. x. x/2 - Asc(x - d') + m Asc (x - d')

As per IS 456, the compression bars with the compressive resistance of the concrete is taken into account shall be the calculated excess multiplied by 1.5 times the modular ratio. The stress here is multiplied by 1.5 times the modular ratio, this should also be multiplied by 1.5 times the modular ratio which is equal to; mAst(d - x)

So here, = mAst (d - x)

This is the equation which is used to find out x. If x is less than or equal to Df, the neutral axis lies within the flange. Now to find out Moment of resistance. If x < xc, it is under reinforced. If x > xc, it is over reinforced. Now, the moment of resistance of the beam here is, the moment of resistance due to the concrete i.e here the whole concrete area , I am taking C_1 as the total compressive force offered by the whole concrete area, which is acting at x/3 from the topmost extreme fibre. T_1 is the corresponding tensile force and (d-x/3) is the corresponding lever arm Z_1. Another thing is, here we are having compression steel i.e C_2. C_2 is the compressive force due to the compressive steel which is acting at d' from the topmost extreme fibre and T_2 is the corresponding tensile force and distance between these two forces is another lever arm Z_2. Now,

$Mr = C_1 Z_1 + C_2 Z_2$

bf . x is the area i.e

C_2 the total compressive force, The total compressive force is stress multiplied by area.

σ cbc' is the stress at the level of compressive steel multiplied with the lever arm which is the moment of resistance.

This is the moment of resistance of the first case, x is less than or equal to Df.

Let us summarize this lecture. In this lecture we have seen, the working stress method and analysis and design of the flanged beams. So here, in this lecture we have seen how to find out the depth of neutral axis i.e critical depth of neutral axis and the actual depth of neutral axis and the moment of resistance of a singly reinforced rectangular beam and we have also worked out certain examples of singly reinforced rectangular beam. In addition to that, we have analyzed the doubly reinforced flanged beam where the neutral axis lies within the flange. In this connection, we need to have certain question i.e determine the expression for the depth of neutral axis for the doubly reinforced flanged beam when the neutral axis lies within the flange and find the corresponding moment of resistance. With regard to the references, we need to have IS 456:2000 Plain and reinforced concrete - code of practice and we have another one book by S.N Sinha i.e Reinforced concrete design. With this we come to the end of this lecture, Thank you!