

Design of Structures II

Lecture 7

Welcome to the UGC lecture series in B.Architecture. Let us have a discussion on Design of Structures II, Unit 1 - lecture 7. In this one, we are going to design the doubly reinforced rectangular section. In the previous lecture, we have seen how to find out the moment of resistance, how to analyze the doubly reinforced rectangular section and how to find out the moment of resistance of the section. Using the expressions of the moment of resistance of the doubly reinforced sections we are going to find out what is the reinforcement required for the doubly reinforced section. This doubly reinforced section normally consists of the reinforcement at the bottom as well as at the top. So, at the bottom it consists of A_{st1} as well as A_{st2} . Total $A_{st} = A_{st1} + A_{st2}$ and at the top it is taken as A_{sc} . Here, how do we find out the reinforcement? In the case of doubly reinforced section, we first need to decide whether the section is a balanced singly reinforced section i.e Design of Doubly reinforced rectangular section. Here our aim is to find out A_{st1} , A_{st2} and A_{sc} .

This is one formula which is used to find out the moment of resistance. Another formula is;

$$M_r = A_{st1} \sigma_{st} (d - x/3) + A_{st2} \sigma_{st} (d - d')$$

Now, to first find out A_{st1} ; Assume our section is balanced singly reinforced section. When you want to find out the balanced singly reinforced section, we need to first find out x_c .

Once we find out x_c , we can find out A_{st1} . Once, you find out moment of resistance of balanced singly reinforced section is given;

This is the moment of resistance of the balanced singly reinforced section. Here, this is the moment of resistance of the singly reinforced section. This is the moment of resistance due to compression reinforcement. Once you find out the moment of resistance of the balanced singly reinforced section, we can easily find out what is the area of A_{st1} required for M_r i.e

$$M_r = A_{st1} \sigma_{st} (d - x_e/3)$$

This is the expression required to find out A_{st1} required for a balanced singly reinforced rectangular section and another one is, it can be found from equating these two things i.e C_1 to T_1 i.e compressive force C_1 to T_1 or by equating C_1 to T_1 at the neutral axis.

Using this expression we can easily find out A_{st1} . Then to find out A_{st2} and A_{sc} . How to find out A_{st2} ? The total moment of resistance is this one, we have found the moment of resistance of the balanced singly reinforced section and total M is given in the problem. So, total Moment;

$$M = M_r + M_1$$

M_1 is the moment of resistance due to the compression reinforcement and due to A_{st2} . So, to find first M_1 i.e excess moment.

$$M_1 = M - M_r$$

So for M_1 , find A_{st2} and A_{sc} . For this excess moment we are going to find out A_{st2} and A_{sc} . So,

$$M_1 = A_{st2} \sigma_{st} (d - d')$$

So from this, this is M_r . If you take this as M , this is M_1 .

Now, to find A_{sc} ; by equating this C_2 and T_2 i.e

$$(1.5m - 1) A_{sc} \sigma_{cbc'} = A_{st2} \sigma_{st}$$

From that, we can easily find out what is A_{sc} .

This is one expression. These are the expressions used to find out (or) we can also find out using;

$$M_1 = (1.5m - 1) A_{sc} \sigma_{cbc'} (d - d')$$

i.e this expression.

This is the formula to find out the area of steel required for the doubly reinforced section. Now, here to design the section.

To find x_c , first we are finding x_c , then by equating C_2 T we are finding A_{st1} . Then we are finding moment of resistance of the balanced section and we are finding the excess moment, $M - M_r$, for the excess moment we are finding A_{st2} and also we are finding A_{sc} by equating the area of equivalent tensile steel to the compressive steel above the neutral axis. Now, we need to design the section. Here, they have given the section i.e the first problem - a doubly reinforced section is given, this is reinforcement at the tension zone. So this is A_{st} which is equal to;

$$A_{st1} + A_{st2}$$

and this is A_{sc} , which is placed at d' from the topmost extreme fibre. Now here, the section is 360 x 750mm i.e

$$b = 360\text{mm}$$

$$d = 750\text{mm}$$

Now, the beam has to resist the bending moment of 300 kNm.

$$M = 300 \text{ kNm}$$

and to find the tensile and compressive steel required for $\sigma_{cbc} = 7$ and $\sigma_{st} = 190$

$$\sigma_{cbc} = 7\text{N/mm}^2$$

$$\sigma_{st} = 190 \text{ N/mm}^2$$

$$d' = 50 \text{ mm}$$

$$A_{st} = A_{sc}$$

Now first to find out x_c .

$$x_c = 247\text{mm}$$

So first by assuming, it is balanced singly reinforced section. So balanced singly reinforced, we have found x_c . Now, to find out A_{st1} for balanced singly reinforced section. Before that, we need to find out M_r of balanced singly reinforced section i.e

$$M_r = 360 \times 247 \times \frac{7}{2} \left(750 - \frac{247}{3} \right)$$

$$= 207.8 \text{ kNm}$$

Now, to find A_{st1} for balanced singly reinforced section. At the neutral axis;

$$C_1 = T_1 \text{ i.e}$$

(or)

$$M_r = A_{st1} \sigma_{st} \left(d - \frac{x_c}{3} \right)$$

This is the moment of resistance of the balanced singly reinforced section. We can also use this expression to find out A_{st1} .

So here by substituting; we get

$$= 1638 \text{ mm}^2$$

Now, to find out A_{st2} and A_{sc} for Excess moment. The excess moment i.e the remaining moment.

$$M_1 = M - M_r$$

M is given in the problem

$$= 300 - 207.8$$

For this excess moment, we are going to design A_{st2} and A_{sc} since it is doubly reinforced section.

$$= 90.2 \text{ kNm}$$

Now, to find out A_{st2} ;

$$M_1 = A_{st2} \sigma_{st} (d - d') = 693.23 \text{ mm}^2$$

But total A_{st} ;

$$A_{st1} + A_{st2}$$

$$= 1638 + 693.23$$

$$= 2331.23 \text{ mm}^2$$

So total A_{st} . Now, to find A_{sc} ;

$$M_1 = (1.5m - 1) A_{sc} \sigma_{cbc'} (d - d')$$

(or)

$$(15m.1) A_{sc} \sigma_{cbc'} = A_{st2} \sigma_{st} = 1242.14 \text{ mm}^2$$

So, that is all about the design of the doubly reinforced rectangular section. So here, we have used one worked example to find out the design of the doubly reinforced rectangular section. So here, they have given the size of the beam and the beam has to resist the moment of 300 kNm and they have asked us to design the reinforcement i.e both tensile and compressive steel reinforcement. So here, the stresses in concrete and steels are given in the problem. So we have started designing the section. First we have found the area of reinforcement required for a balanced singly reinforced section. At neutral axis, $C_1 = T_1$ (or) the moment of resistance;

$$M_r = A_{st1} \sigma_{st} (d - x_c/3)$$

we have found the A_{st1} required for it and we have found the excess moment i.e The excess moment is the given moment minus the moment of resistance of the balanced singly reinforced section i.e 90.2. For the 90.2 kNm, we have found A_{st2} and we have found the total A_{st} and for finding out A_{sc} , we have equated the moment of area of the compression side to the moment of area at the tension side (or) we can use this expression to find out the A_{sc} required for it i.e 1 - 4.14. So here, if you want to find out i.e here this is the total A_{st} , we can assume the diameter of the reinforcement and find out for example, as in diameter. 25mm diameter, we can find out number of bars required for it.

We can easily provide how many numbers of reinforcements required for it. At the same time here also, assume the diameter of reinforcement. I can also have 16mm diameter bar. So, number of bars is equal.

So, we can easily find out how many numbers we need to provide. So, this is the rectangular section. We can easily provide the reinforcement required for it. If this is A_{st} , this is A_{st} and this is A_{sc} and this is the size of the section;

Now, let us summarize this lecture. In this lecture we have discussed about the working stress design method of a doubly reinforced rectangular section. We have worked out one example, in this example they have given the size of section, as well as the stresses in the concrete and steel. They have asked us to design the area of reinforcement required for it. Since, it is a doubly reinforced section, we need to find out the area of steel required at the tension zone as well as at the compression zone. In the case of tension zones, it consists of A_{st1} and A_{st2} . A_{st1} is used to find out, the area of reinforcement required for the balanced singly reinforced section and A_{st2} is used to balance the reinforcement which is provided at the compression zone. For finding out A_{st2} we are finding out the excess moment i.e excess moment which is found from the total moment minus the moment of resistance of the balanced singly reinforced section. For that excess moment, we are providing the additional tensile reinforcement at the bottom, that is taken as A_{st2} and for balancing that one, we are provided with the reinforcement at the

top. For M1 also, we have provided the reinforcement at the compression zone i.e Asc. This, Ast₂ is used to balance this Asc, that is called as additional tensile reinforcement i.e Ast₂. So the total Ast is normally called as Ast = Ast₁ + Ast₂

This is the way we have designed the doubly reinforced rectangular section by using one worked example and the questions.

In the question here as well, they have asked us to design the section by giving a size of section, as well as the external moment carrying capacity of the beam. So here, the doubly reinforced concrete beam, 250mm wide and 600mm deep overall, has to resist an external bending moment of 95 kNm. Find the amount of tensile steel and compressive steel required. If covered to the centre of the steel on both sides is 50mm. Here, they have given; $\sigma_{cbc} = 5 \text{ N/mm}^2$ and $\sigma_{st} = 140 \text{ N/mm}^2$. So here also, first we need to assume our section is a balanced singly reinforced section. We need to first find out what is the Ast required for the balanced singly reinforced section i.e at the neutral axis, $C_1 = T_1$. Find out Area of reinforcement i.e Ast required for the balanced singly reinforced section and we need to find out what is the moment of resistance of the balanced singly reinforced section. Then, normally we know that the moment of resistance of the doubly reinforced section is the moment of resistance of the balanced singly reinforced section plus the moment of resistance due to the compression reinforcement. So here, we need to first find out what is the moment of resistance of the balanced singly reinforced section. The total moment of the problem is given in the problem i.e the moment of doubly reinforced section which must be equal to the moment of resistance of the balanced singly reinforced section plus the moment which is provided for the additional tensile reinforcement and the compression reinforcement. Again we need to find out the excess moment. Excess moment, M₁ equal to the total moment that is given in the problem minus the moment of resistance of the balanced singly reinforced section. So for that excess moment M₁, we need to find out what is Ast₂. Then for finding out Ast by equating the moment of area on compression side which is equal to the moment of area of tension side i.e

$$(1.5m - 1) A_{sc} \sigma_{cbc} = A_{st2} \sigma_{st}$$

So, Asc we can easily find out from this expression. So, totally we can find out Ast₁, Ast₂ and Asc. So, here total Ast = Ast₁ + Ast₂ and Asc we have found. So this is the question. With regard to the references, we need to refer; IS456:2000 i.e Plain and reinforced concrete - Code of Practice and we can also refer a book in order to work out more examples i.e Reinforced concrete design by S.N Sinha. Let us come to the end of this lecture, thank you!