Design of Structures II

Lecture 06

Welcome to the UGC lecture series in B.Architecture. Let's have a look on a 'Design of structures II'. In the previous lecture we have seen how to analyze the doubly reinforced rectangular section and how to find out the moment of resistance of the respective sessions. In this schedule we are going to work out some problems to find out the moment of resistance of the given section. We are going to do two problems, one is under reinforced section, the second is over reinforced section in the case of doubly reinforced section.

The first problem, a beam 300 by 600mm effective, since it is doubly reinforced section we need to also have the reinforcement at the top. But this is the reinforcement which is placed at d' from the topmost extreme fibre. This is the area of reinforcement which is placed at a deep 600mm from the topmost extreme fibre and this is Asc and in this problem, the size of the beam is 600mm by 300mm and is provided by tensile and compressive reinforcement of 1256mm ^{2} each i.e Ast = 1256mm ^{2}, Asc is also equal to 1256 mm ^{2} i.e given in the problem;

b = 300mm
d = 600mm
Ast = 1256mm ^{2}
Asc = 1256mm ^{2}

and the compressive steel is placed at 40mm from the top edge.

d' = 40mm
σ cbc = 7 N/mm ^{2}
σ st = 190 N/mm ^{2}
Mr = ?

Here, we are having two equations. One is;

$$Mr = \frac{bx\ \sigma\ cbc}{2\ (d - x/3) + (1.5m - 1) Asc\ \sigma cbc\ \ (d - d')}$$

Mr = Ast1 σst (d.x/3) + Ast2 σst (d - d')

These are two equations which are used to find out the moment of resistance of the beam. Here, totally it is given in the problem but here we need Ast1 and Ast2 separately. This is the expression which is used to find out the moment of resistance of under reinforced section, this is for over reinforced section. We can use this expression to find out the moment of resistance of under reinforced section, once you find out σ cbc actual at the depth of the neutral axis. We can substitute here, we can find out the moment of resistance using this expression for the under reinforced section. In case of singly reinforced section also we can use. In the case of singly reinforced section;

Mr = bx σcbc (d - x/3)

This is for over reinforced section.

Mr = Ast σst (d - x/3)

This is for under reinforced section.

We can also use this expression to find out the moment of resistance for under reinforced section. Once you find out σ cbc actual at x from the neutral axis. Once you substitute here, we can find out the moment of resistance of this expression for under reinforced section. You can also check that answer with the actual expression of under reinforced section, both will be the same. Here also, I am going to use the expression to find out the moment of resistance for both under and over reinforced section using the respective σ cbc. So here, first I am going to find out Xc. We need to first find out Xc. First, we need to find out whether our section is under reinforced or over reinforced section. So, first we need to find out the critical depth of neutral axis i.e

$$\frac{\sigma cbc}{\sigma cbc} = \frac{Xc}{d - Xc}$$

So, σ cbc is 7 and σ st is 190.

$$\frac{7}{190/13{-}33} = \frac{Xc}{600 - Xc}$$

Where, m $= \dfrac{280}{34bc}$

X_C = 197. 61mm

Now, we need to find out Xc. Now to find out x, the actual depth of neutral axis.

$$\frac{bx^2}{2} + (1.5m - 1)Asc\,(x - d) = m_{Ast}(d - x)$$

This is the expression which is used to find out the actual depth of neutral axis for the section. I am going to substitute the expression.

$$\frac{300x^2}{2} + (1.5 \times 13.33 - 1)\,1256(x - 40) = 13.33 \times 1256(600 - x)$$

So, we get the quadratic equation;

$x^2 + 271.6x - 71741 = 0$

$x = 164.778mm$

This is x. x is 197.61. X is less than Xc, it is under reinforced section. In the case of under reinforced sections I have told you that the steel is test to its maximum permissible value i.e

σst is known = 190 N/mm ^{2}

It reaches maximum permissible limit. But the stress in concrete is stressed below its permissible value. I am going to find out the actual σ cbc @ x from the neutral axis.

$$\frac{\sigma cbc\ (ac)}{\sigma st\,/\,m} = \frac{x}{d-x}$$

For my x, I am going to find out σ cbc by substituting;

$$\frac{\sigma cbc}{190/13.33} = \frac{164.778}{(600 - 164.778)}$$

σ cbc (ac)= 5.4 N/mm ^{2} which is less than 7 N/mm ^{2} . This is the permissible limit. Normally, in the case of under reinforced section. Concrete is stressed below its permissible value, that's why I have found actual stress at the topmost extreme fibre which is 5.4 which is less than 7, that is given in the problem 7. So, here the moment of resistance used in this expression;

Mr = $\frac{bx\,\sigma cbc}{2\,(d.x/3)}$ + (1.5m - 1) Asc σ cbc' (d-d') This is the formula. I need to first find out what is σ cbc'. So I know the actual σ cbc. I need to find out what is σ cbc at the depth of d' at the topmost extreme fibre. So, here when you have the stress diagram, this is σ cbc (ac) and this is σ cbc' which is at a distance (x - d') at the topmost and this is at a distance x.

σ cbc/ x = $\frac{\sigma cbc'}{x - d}$ Suppose, σ cbc' is σ cbc/x (x - d')

= 4.08 N/mm ²

I have found the σ cbc. Now I am substituting all the expressions in this equation i.e

Mr = $\frac{300 \, x \, 164.778 \, x \, 5.4}{2}\left(\frac{600 - 164.778}{3}\right)$ + (1.5x 13.33 - 1) 1256 x 4.08(600 - 40)

= 127.26 kNm

This is the answer for the first question. So, here I have used this expression to find out the moment of resistance of the under reinforced section by calculating the actual cbc at the topmost extreme fibre at the depth of x from the neutral axis. At the depth of x from the neutral axis, the actual σ cbc should be less than the permissible σ cbc in the case of under reinforced section. Since, in the case of under reinforced section steel is stressed to its maximum permissible stress where the concrete is stressed below its permissible value. That's how I have found actual σcbc and I have also found out σ cbc' and I have used this expression to find out the moment of resistance for the under reinforced section.

Next one, An RC beam 250 by 550mm overall. $\frac{250mm}{550mm}$ overall. D is given in the problem, b = 250. Which is reinforced of four numbers of 25mm diameter on tension side. Four numbers of 25mm on tension side, 4 x 25mm diameter, Ast. Three numbers of 22mm diameter bar, 3 x 22mm diameter, Asc which is on the compression side. The bars are at 50 and 40mm centres from bottom and top edges of the beam respectively. Top = 40mm d', this is at a distance of 50mm from the bottom. So, the effective depth of beam is;

d= 500 - 50 = 450mm

So, d' = 40mm

You can calculate moment of resistance, σ cbc = 5 N/mm ², σ st = 140 N/mm ². Find the moment of resistance of the beam. So, here also first to find Xc.

$$\frac{Xc}{d} - X_c = \frac{\sigma cbc}{\sigma st / m}$$

$$\frac{Xc}{450} - X_c = \frac{5}{140/18.66}$$

Xc = 200mm

Then, to find x.

$bx^2 + (1.5m - 1)\,Asc\,(x - d') = m_{Ast}\,(d - x)$

So by substituting all the components,

$$250 - \frac{x^2}{2} + (1.5 \times 13.33 - 1)\,\frac{3\pi}{4 \times 22^2}(x - 4)$$

$$= \frac{13.33 \times 4 \times \pi}{4\,(25)^2\,(600 - x)}$$

$x^2 + 539.18x - 1539x\ 2 = 0$

x = 206.41mm

So here, x > xc, it is an over reinforced section. In the case of an over reinforced section, the concrete is stressed to its maximum permissible value where the steel is stressed to its permissible value. So, we know;

σ cbc is known = 5 N/mm^2

We need to find out, σ cbc' i.e at the level of d' at the topmost extreme fibre i.e a depth of (x - d') from the neutral axis. This is at x. At x it is 5N/mm^2.

$$\frac{\sigma cbc}{x} = \frac{\sigma cbc'}{(x - d')}$$

$$\sigma\ cbc' = \frac{\sigma cbc}{x\,(x - d')}$$

After substituting all these values;

$= 4\ N/mm^2$

Now, we need to find out the moment of resistance of the section.

Mr for over reinforced section;

$$Mr = \frac{bx\sigma cbc}{2}(d - x/3) + (1.5m - 1)\ Asc\ \sigma\ cbc'(d-d')$$

After substituting all these values, we can get;

$$= \frac{250\ x\ 200.41\ x\ 5}{2}\left(\frac{450 - 206.41}{3}\right) + (1.5 \text{ x } 13.33 - 1)\frac{3\pi}{4} \text{ x } 22^2 \text{ x } 4(450 - 40)$$

$= 112.14\ k_n m$

This the way of calculating the moment of resistance of the respective sections. One is, if it is an under reinforced section; for x < xc an under reinforced section;

$$Mr = Ast,\ \sigma\ st\ (d - x/3) + Ast_2\ \sigma\ st(\ d - d')$$

This is one expression. If x > xc, it is an over reinforced section. So moment of resistance of the over reinforced section is;

$$Mr = \frac{bx\ \sigma\ cbc}{2}(d - x/3) + (1.5m - 1)\ Asc\ \sigma\ cbc'\ (d - d')$$

So here, in the case of under reinforced sections, Ast1 and Ast2 are not separately given in the problem, we can use this expression to find out σ cbc (ac) for x. By substituting here, we can easily find out what is the moment of reinforcement for the under reinforced section. So here, we can use this as an expression to find out moment of under reinforced section and by calculating actual σ cbc. This is the one we have found. We have worked out two problems. Thus, we complete the analysis of doubly reinforced rectangular section.

Let us summarize this lecture. In this lecture we have seen some worked examples and analysis of doubly reinforced section. In this, we have found actual depth of neutral axis and critical depth of neutral axis. Then comparing the actual depth of neutral axis with the critical depth of neutral axis, we have found whether our section is an under reinforced or an over reinforced section. Then, we have found the moment of resistance of the respective section using the formulas.

Now, the Questions. Here also, I have given a couple of workable examples. We can find out the moment of resistance of the doubly reinforced beam of size 230 x 500mm overall in section which is reinforced with four 20mm diameter bars on tension side and three 20mm diameter bars on compression side. The bars are at 50 and 40mm centres from the bottom and top edges of the beam respectively. So, the σ cbc and σ st is 7 N/mm ^{2} and 190 N/mm ^{2}. So, we can find out the moment of resistance of this section, from this we can easily find out the moment of resistance of the doubly reinforced section. For this lecture, we can refer; IS 456:2000 as an IS code of provision i.e Plain and reinforced concrete Code of practice. Then we can also refer a book by S.N Sinha i.e Reinforced concrete design. Let us conclude this lecture, Thank You!