Design of Structures II

Lecture 5

Welcome to UGC lecture series in B.Architecture. Now we will have a look at Design of Structures II, lecture 5. In the previous lecture, we have seen or worked on some examples to find out the moment of resistance of singly reinforced section and also using the moment of resistance equations, we have designed the size of section as well as the Area of reinforcement required for it. Now, we move on to Doubly reinforced rectangular section. What do you mean by doubly reinforced rectangular section? The section which is reinforced with the reinforcement or provided with the reinforcement both the top and the bottom i.e both at the compression and tensile zone is known as doubly reinforced rectangular section. Under what circumstances the doubly reinforced rectangular sections are designed? Certain circumstances due to structural or architectural recommendations, the size of sections can be reduced. Due to this, the amount of concrete area above the neutral axis is completely reduced in order to compensate the stress or in order to supplement the stress in the concrete above the neutral axis. We are in the place of providing the reinforcement at the compression zone. When you provide the reinforcement a the compression zone, the section will become doubly reinforced rectangular section. Due to the reinforcement of the doubly reinforced rectangular section, the moment of resistance can be increased without altering the dimensions of the section and this is the doubly reinforced rectangular section. So, due to structural or architectural considerations, the section will have insufficient area of concrete to provide for compressive stresses when you reduce the size of concrete. So, steel is placed at the part of compression zone in order to supplement the concrete in resisting the compressive stresses. So, when you provide the reinforcement at the top i.e at the compression zone, the section will become doubly reinforced rectangular section. So, here the moment of resistance can be increased without altering the dimensions of the section. But what about the moment of resistance of the doubly reinforced section? The moment of resistance of the doubly reinforced section is the moment of resistance of the singly reinforced section plus the moment of resistance due to the compression reinforcement at the topmost extreme fibre. So, here also we have three types of sections; one is balanced sections, under reinforced section and over reinforced section as in the case of the singly reinforced rectangular section. We have already seen what do you mean by the balanced sections. The reinforced sections are designed based on the assumptions that the stresses in concrete and steel reach their maximum permissible stresses one and at the same time. Both the materials are stressed to their fullest allowable stresses at the same time, that section is called balanced section. In the case of under reinforced section if you use less steel than what is required for the balanced section, the section will become under reinforced. In such conditions, the steel is stressed to its maximum permissible value while the concrete is stressed below the permissible stress. Under such conditions, the section will fail initially due to overstress in steel. So, the type of failure that occurs in under reinforced section is; Tensile failure or ductile failure. In the case of under reinforced section, the depth of neutral axis, which is normally going to be i.e the neutral axis is normally going to be situated in the centre of gravity of this section. In this section, the neutral axis which is mostly upward, when you compare the actual depth of neutral axis of the under reinforced section with the balanced sections, we have X is less than Xc. When the neutral axis, the actual depth, the actual neutral axis moves upwards. When you come to the over reinforced sections, more steel than what is required for the balanced section is used, will be called as the over reinforced section. In such conditions, the concrete is stressed to its maximum permissible value while the steel is stressed below its permissible value. Under such conditions, the section will fail initially due to over stress in concrete. So, here the neutral axis moves downward. So, here X is greater than Xc, we get it over reinforced section.

Now, we move on to, how to find out the moment of resistance of the doubly reinforced rectangular section. How to analyze this section? How to find out the moment of resistance? As in the case of singly reinforced section, here also we are going to analyze the doubly reinforced section, we are going to find out the resistance which can be used to find the moment carrying capacity of the respective sections. Once we find out Xc and you find out X for the respective section, when you compare X with Xc, we can finalize whether it is under reinforced section, over reinforced section or balanced section. Then we are going to find out the moment carrying capacity of the respective section. Using these two, moment of resistance equation, we are going to design the section, we are going to find out the size of the section, we are going to find out what are the area of steel required for resisting the moment of resistance at the bottom as well as the top i.e total Ast required for the beam as well as the total Asc which is required to resist the compressive stresses at the top. Now, how to find out the moment of resistance? Here, a doubly reinforced rectangular section. Since it is a doubly reinforced section, the steel must be provided both at the top as well as at the bottom. This is total Ast required for the section and this is the area of compression reinforcement which is denoted by Asc, which is placed at d' from the top most extreme fibre and b is the breadth of the beam and small d is the effective depth of the beam. This is a doubly reinforced rectangular section. Here, again I am taking this as the neutral axis of the given section. This one is the strain diagram i.e maximum strain at the topmost extreme fibre is Є C and this is Є T, the tension reinforcement. This is a strain diagram and this is a stress diagram. So maximum stress at the topmost extreme fibre is σ cbc and the stress or the distress d' from the, is called as σ cbc' and T is the tensile stress, tensile force and the equivalent stress diagram, here it is. This is the equivalent stress diagram by transferring the area of steel into the equivalent area of steel. So σ cbc is the stress at the topmost extreme fibre, σ st/m is the stress in concrete surrounding tensile steel, this we have already derived. Now, here this is, you take this is as σ cbc' which is at a distance, d' from the topmost extreme fibre. Now, to find out Xc, you take this as Xc and this as d - Xc. You take this as the equivalent stress diagram for the balanced section. So, Xc/ σ cbc; at a depth of Xc from the neutral axis, the maximum stress is σ cbc. At a depth of Xc from the neutral axis, the stress in the concrete surrounding tensile steel is σ st/m. This is the expression which is used to find out, critical depth of neutral axis. Then to find out, X, actual depth of neutral axis. To find out actual depth. Normally, here in the case of working stress method, while you find out the actual depth of neutral axis for the unbalanced section, the neutral axis is going to be situated at the central gravity of the section. To find out the depth of neutral axis, we are going to equate i.e by equating moment of area of compression side to the moment of area of tensile side/ equivalent moment of concrete area of tensile side. So, moment of area of compression side = total area i.e b x X is the area. I have already explained the moment of area to you. Moment of area = area x Distance of C.G of area from the neutral axis. This is the moment of area. So, the distance of C.G of this total concrete from the neutral axis, here it is X/2. You take, this is the equivalent stress diagram of unbalanced section. You take this as X. Now, b x X/2 +/-. Since the steel is placed at the compression zone, we need to first deduct the moment of area due to the compression steel when here we have considered whole concrete area, first we need to deduct the moment of area due to the compression steel. The moment of area is area of the steel Asc x distance C.G of the steel from the neutral axis i.e X - d'. Then we add, equivalent concrete area of the compression steel i.e m x Asc(x - d'). This is the equivalent concrete area. This is the area of the compression steel, total concrete area, then we deduct the moment of area of the compression steel, then we provide equivalent concrete area of the compression steel. So, the moment of area of the compression side is equal to the equivalent moment of area of concrete on the tensile side which is equal to the moment of area due to whole concrete above the neutral axis - the moment of area due to the compression steel. Then I add moment of equivalent concrete area of the compression steel i.e m x Asc is the equivalent concrete area; x - d' is the distance of C.G of steel from the neutral axis. As per IS 456 where the compressive resistance where the compression in bars where the compressive resistance of the concrete is taken into account shall be the calculated stress multiplied by 1.5 times the modular ratio. So, σ sc = 1.5 times the calculated state at the compression zone. So, if it is multiplied by 1.5 times with the modular ratio along with the stress. This area should also be again multiplied by 1.5 times. So, $bx^2/2$ - Asc (x - d') + 1.5m Asc (x - d'), this is the moment of area of concrete above the whole neutral axis. Then, I have deducted the moment of area due to the compressive steel, then I have added equivalent concrete area of the compression steel. So, here compression in bars where the compressive resistance of the concrete is taken into account, shall be the calculated stress multiplied by 1.5 times the modular ratio. Here, the stress is multiplied by 1.5 times the modular ratio, then this area should also be multiplied by 1.5. So, here which is equal to, equal moment of the concrete area of the tension steel. So, here C_1 be the total compressive force which is offered by the whole concrete area above the neutral axis, which is normally acting at x/3 from the topmost extreme fibre and t1 be the corresponding tensile force and the distance between these two forces is called the lever arm is at 1 and this is d - x/3 and here again, c_2 be another compressive force which is offered by the compression steel which is placed at d' from the topmost extreme fibre and to balance the force there is another tensile force d_2. The distance between d_2 and c2 is again called a lever arm Z_2 i.e (d - d'). So, here this is the moment of area of the compression side which must be equal to, equivalent moment of concrete area of the tensile side, that is equal to m x Ast (d - x), where m is the modular ratio, a is the area of steel, when Ast is multiplied by the modular ratio, it becomes equivalent concrete area surrounding tensile steel and (d - x) is the distance of C.G of the tensile steel from the neutral axis. So, here:

$bx^2/2 + (1.5m - 1)\ Asc(x-d') = mAst(d-x)$,

this is the expression which is used to find out X for the doubly reinforced section. So, we have found Xc and X. Now we are going to find out the moment of resistance, the moment of resistance of the beam. Now, we will compare; if X is less than Xc, it is an under reinforced section. The moment of resistance, Mr = T x Z, this we already know and in case of under reinforced section, T is stressed to its maximum permissible point while the concrete is stressed below the maximum permissible value. The section which normally fails initially due to overstress in steel. The moment of resistance which is;

$$T \times Z = T_1 Z_1 + T_2 Z_2 = Ast_1\ \sigma\ st\ (d-x/3) + Ast_2\ \sigma\ st\ (d - d')$$

Moment of resistance is, the moment of resistance of a singly reinforced section plus additional moment of resistance due to the compression steel. If X>Xc; it is over reinforced section. The moment of resistance for an over reinforced section is;

$$Mr = C \times Z = C_1 Z_2 + C_2 Z_2 = b.x\ \sigma\ cbc/2\ (d - x/3),$$

this is the moment of resistance of singly reinforced section + the moment of resistance due to C_2 i.e Area of compression steel i.e Area of steel here is equivalent area of concrete surrounding tensile steel i.e (1.5m - 1) Asc.σ cbc' (d - d'), this is the corresponding stress at the level of compressive steel, this is the lever arm for the compression steel. M = C x Z = bx σ cbc/2(d-x/3) + (1.5m - 1)Asc σ cbc' (d - d'). This is the part we have derived.

We have started analysing the doubly reinforced rectangular section. In this also, we have three sections, one is balanced section, under reinforced section, over reinforced section. This is the rectangular section which has reinforced both the tension and compression zone. Area of reinforcement of the tension zone is Ast, Area of reinforcement of the compression zone is Asc. So, here Ast which is normally

$$Ast_1 + Ast2.\ Ast_1$$

is used to resist the total compressive force of the total concrete area and Ast2 is the additional reinforcement to compensate the area of compression in steel. In this case, we need to find out Ast_1, Ast2 and Asc. So,

total Ast is Ast_1 + Ast2

So, here this is the strain diagram. The maximum strain is at the topmost extreme fibre, ЄC. Here, it is ϵ_t and this is the stress diagram. Maximum stress is at the topmost extreme fibre σ cbc and the stress at the level of compressive Steel is σ cbc'. This is the easiest equivalent stress diagram converting the area of steel into the equivalent area of concrete i.e σ cbc is the maximum stress at the topmost extreme fibre at the place of concrete surrounding tensile steel is σ st/m and σ cbc' is the stress at the compressive steel i.e σ cbc' and here C_1 be the total compressive force which is offered by the whole concrete area above the neutral axis, C_2 be the total compressive force offered by the compression reinforcement which is acting at d' at the topmost extreme fibre. T_1 is the tensile force which is used to balance C_1. T2 be the total tensile force which is used to balance C_2 and the resultant distance between C_1 and T_1 is Lever arm Z_1 i.e (d-x/3) where C_1 is the total compression offered by the whole concrete area above the neutral axis which is acting at x/3 from the topmost extreme fibre and the distance between C_2 and T2 which is another lever Arm which is (d-d') where d_2 is the tensile force which is used to compensate C_2, where $C_2$is the compressive force which is due to the area of compression steel which is acting at d' from the topmost extreme fibre. So this is the, stress strain diagram. From the stress strain diagram, we have found the value of Xc. Xc by equating from the similar triangle property of the balanced section, we have found;

Xc/σ cbc = (d- Xc/σ st/m)

This is the equation which is used to find out Xc. Then, to find out Xc in the case of unbalanced section since the neutral axis is situated by the centre of gravity of the section, in order to find out X, we are going to equate the moment of area on the compression side to the moment of equivalent moment of concrete area of the tensile side. So, here bx^2 is the total concrete area above the neutral axis into the distance i.e the area of the whole concrete x Distance of C.G of area from the neutral axis. Since the steel is placed at the compression zone, I am deducting the moment of area of the compression steel, area of the compression steel x distance of C.G of the compression steel from the neutral axis and I am adding the equivalent area of the compression steel i.e m x As x (x-d'). So, the compression in bars as per IS 456 or the compressive resistance of the concrete is taken into account, shall be the calculated stress, will be 1. times the modular ratio. So, here the stress has been multiplied by 1.5 times with the modular ratio. This area should also be multiplied by 1.5. After simplifying this one,

$$bx^2/2 + (1.5m - 1)Asc\,(x - d')$$

which must be equal to mAst(d-x). This is the expression for finding out the actual depth of neutral axis. Using this, we come conclude that, when X is less than Xc, it is an under reinforced section. So, the moment of resistance normally, T x Z where ;

$$T_1 \times Z_1 \text{ and } T2 \times Z_2 = Ast_1\, \sigma\, st(d - x/3)$$

is the moment of resistance of balance area of singly reinforced section;

$$+ Ast2\, \sigma\, st\,(d - d')$$

is the moment of resistance to balance the compression reinforcement due to the additional tensile reinforcement. If X is greater than X, we conclude that it is an over reinforced section. So,

$$Mr = C \times Z = C_1 \times Z_1 + C_2\, Z_2,$$

this is the moment of resistance due to the whole concrete area above the neutral axis and this is the moment of resistance due to the compression steel. Using this, we can easily find out the moment of resistance.

Now, let us have a look at the summary of this lecture. So, we have analyzed, the doubly reinforced rectangular section by working stress design method and in this case, we have found Xc i.e the critical depth of neutral axis and we have found the actual depth of neutral axis of the section. By using and by comparing these sections, we have finalized whether the sections are under reinforced or over reinforced and based on the conclusions, we have found the moment of resistance of the respective sections i.e under reinforced and over reinforced and balanced section. Questions; In this lecture, Under what circumstances doubly reinforced sections are designed? We have discussed this in detail. Second one, Determine the moment of resistance of a doubly reinforced rectangular section. References for this lecture are; IS 456:2000 Plain and reinforced concrete code of practice, this is the IS codal provision. Then a detailed book i.e written by S.N Sinha on Reinforced concrete design. With this we come to the end of this lecture, Thank you!