Design of Structures II

Lecture 4

Welcome to UGC lecture series in B.Architecture. Now we are going to have lecture 4 on a Design of Structures II. In lecture 3, we have seen how to find out the moment of resistance of a singly reinforced rectangular beam, from that using that expression, we are going to design the size of section, as well as reinforcement record for it. This is the expression we have derived i.e the moment of resistance of the section. To find out the moment of resistance , we have first derived the expression for finding out the critical depth of neutral axis Xc and actual depth of neutral axis of this section. By comparing these two X's, we are finalizing our section i.e if X is less than Xc, it is under-reinforced section. Then, the moment of resistance with respect to a section is T x Z i.eAst sigmast (d-x/3) and X is greater than Xc, it is concluded as over-reinforced section. So, C x z = bx sigmacbc/2(d-x/3). Now we are going to solve some problems.

The first one; an RC beam 300 mm wide and 700 mm deep overall, is reinforced with 3 Nos. of 20 mm diameter bars. The centre of bars are 50mm from the underside of the beam. The maximum stresses are not exceed 7 N/mm^2 for concrete and 190 N/mm^2 for steel. Find the moment of resistance and safety distributed load the beam can carry. The span is 6m. Now for finding out the moment of resistance of this problem, we are able to find out what is the safety distributor load by deducting self weight of the beam i.e the actual safely distributed load the beam can carry. So here, the problem is; b = 300mm and overall depth is 700mm, which is reinforced with 3 nos of 20mm diameter bar i.eAst = 3nos of 20 mm diameter and the center of bars are 50mm from the underside of the beam i.e the center of the bars are 50mm from the underside of the beam. So, d = 700 - 50 = 650mm and the maximum stresses are not to exceed, sigmacbc = 7 N/mm^2 in concrete and 190 N/mm^2 for steel. sigmast = 190 N/mm^2 and find the moment of resistance of the beam. They are also asking us to find out what is the load of the beam. So, here this is a singly reinforced, from the figure we can understand it is the singly reinforced rectangular beam since the steel is, since the beam is reinforced with the reinforcement at the tension zone only. So, it is a singly reinforced beam. So, in order to find out the moment of resistance of singly reinforced beam, we have three sections; one is balanced section, another is under-reinforced section, third one is over-reinforced section. So, based on the respective sections, we are going to use the respective equation to find out the moment of resistance. To find out the first moment of resistance of the respective section, first find out Xci.e the critical depth of neutral axis. The critical depth of neutral axis i.eXc/sigmacbc = d-xc/ sigmast/m. This is the expression to find out Xc. Here, Ast is given in the problem i.e three numbers of 20mm diameter bar = 3 x PI/4$(20)^2$ = 942 mm^2. Now, here Xc/ sigmacbc is 7, d = 650 - Xc/ sigma St i.e (190 where m is equal to the modular ratio, this I already told you that in the working stress method, they are using well known mechanics i.e modular ratio

= 280/ 3sigmacbc; 280/3 x 7 = 13.33)

So, from that what we can easily find out what is Xc. Xc is 214.08mm. This is the Xc of our problem. Next, we find out what is X. We have already derived an expression to find out the actual depth of the neutral axis i.e$bx^2/2 = mAst(d - x)$. Now, we are going to substitute the parameters i.e

$$300\ x^2/2 = 13.33 \times 942\ (d \text{ is } 650 - x),$$

x is the term we need to find out. After simplifying this expression, we get a quadratic equation, we need to find out the root i.e a positive root = 195.115mm. So here after finding out the X, we need to compare it to Xc. So, here X <Xc, it is under reinforced section. We need to find out the resistance of the under reinforced section i.e

$$Mr = T \times Z = Ast\ sigmaSt(d - x/3)$$

We need to substitute the values

$$= 942 \times 7(d \text{ is } 650 - X \text{ is } 195.115mm) = 104.7kNm$$

This is the moment of resistance. Using the moment of resistance we are going to find out what is the safely uniformly distributed load. Before it, by equating, this is the internal moment to the external moment M i.e$wl^2/8$, where:

$$Mr = 104.7 \text{ which is } = w \times \text{span is } 6^2/8. \text{ Where } W = 23.27\ kN/m$$

This is the uniformly distributed load, we have found which includes the self weight of the beam. But they have asked us the uniformly distributed load from this we need to deduct the self weight of the beam. Self weight of the beam is normally found from the size of beam = 0.3 x 0.7 x 25. 25 is the unit weight of concrete. 25 kN/m^3 i.e the unit weight of concrete which is equal to 5.25 kN/m. So, the safely distributed load which excludes the self weight of the beam is;

$$W = 23.27 - 5.25 = 18\ kN/m$$

This is the safely distributed load for the first problem. Now, we have found , the first problem they asked us to find the moment of resistance of the beam before that we need to find out what is Xc, what is X, then compare X with Xc, we need to find their respective section i.e here it is x <Xc, it is under reinforced section. The moment of resistance of the respection section i.e

$$T \times Z \text{ i.e} Ast\ sigmast\ (d-x/3) \text{ i.e } 104.7\ kN/m$$

From that we find out the safely distributed load. This is the moment of resistance, that is the internal moment. The external moment due to the uniformly distributed load is $wx^2/8$ for the simply supported beam. From that we have found, w = 23.27 kN/m, from this we are deducting the self weight of the beam i.e 0.3 x 0.7 x 25, the respective

unit weight = 5.25. So, 23.27 - 5.27 = 18 kN/m,

this is the safely distributed load. Second problem, An RC beam 250 x 500mm overall in section, 250/500mm overall. This is 250 and this is 500, overall i.e d = 500; b = 250, is reinforced with four 22mm diameter bars. Four numbers of 22mm diameter bars i.e 4 x 22mm, at the bottom with the cover of 40mm. So, the cover for the reinforcement is 40mm and we can easily find out what is the effective depth. d = 500 - 40 = 460mm, this is effective depth. Effective depth means the depth from the topmost extreme fibre to the centre of reinforcement, that is the effective depth of beam. The effective span of the beam is given i.eleff= 5mm. Find the concentrated load the beam can support, in the first problem they had asked uniformly distributed load, in the second problem they have asked the concentrated load. Concentrated load we can find out from the moment of resistance. Take sigmacbc= 5 N/mm ^{2}, sigmast = 190 N/mm ^{2}. sigmacbc = 5 N/mm ^{2}, sigmast = 190 N/mm ^{2}, this is the problem they have given. So, first find out whether our section is under reinforced or over reinforced, depending upon the section we are going to find out the moment of resistance. Using the moment of resistance by equating the moment of resistance to the external bending moment, we are going to find out what is the concentrated load. First to find out, Xc, Critical depth of neutral axis i.eXc/sigmacbc = d - Xc/ sigmast/m, this is the expression to find out the value of Xc. So Xc divided by sigmacbc is 5. d is 460 - Xc/ sigmast is 190; m is the modular ratio 280/3sigmacbc = 18.67. Therefore,

Xc/5 = 460- Xc/ 190/18.67

From that Xc = 56.44 mm. Then, we need to find out X, X is found from the formula; bx ^{2}/2 = mAst (d-x). By substituting the parameters, it is:

250 x X ^{2}/2 = 18.66 x Ast is 4 x 22 ^{2} = 1520 mm ^{2}; 18.67 x 1520 x (460 - x)

So from that, we have found x as 230 mm. So by comparing X with Xc, here; X >Xc, it is over reinforced section. In the first problem, we saw under reinforced section, but here this problem falls under over reinforced section. So, the moment of resistance of the over reinforced section is C x Z = bx sigmacbc/2 (d - x/3) = 250 X 230 x 5/2 (460 - 230/2) = 58.05 kN/m. This is the moment. Now, self weight of the beam. I am going to find out what is the concentrated load but Self weight of the beam is the uniformly distributed load that I am going to deduct this self weight of the beam from the moment of resistance i.e External load which is uniformly distributed

= 0.25 x 0.5 x 25 = 3.125 kN/m.

By equating the moment of resistance to the external bending moment i.eWl ^{2}/8 i.e 58.05 = w x 5 ^{2}/8; the moment due to self weighti.e 3.125 x 5 ^{2}/8 = 9.765 kN/m. This is the moment. Net moment, I am going to deduct the moment of self weight from the total moment i.e 58.05 - 9.765 = 48.285 kN/m. This is net moment after deducting the self weight of the beam. Now, using this moment, now I am going to find out what is the concentrated load on this beam, I am assuming that the concentrated load is acting at the center of the beam. The moment is Wl/4. So, 5 x W/4 which is

= 48.225 where W = 38.62 8 kN.

This is the concentrated load acting on the beam. In the last problem we found, the uniformly distributed load i.e 18 kN/m after deducting self weight. In this problem, they have asked us to find out the concentrated load. So, once we found out the moment of resistance, we found out the self weight due to the self weight, we found out the moment then that moment has to be deducted from the total moment i.e 48.225, from this moment I have found out the concentrated load acting on the beam. This is the way to analyze the section.

Next, to design the section. How to design the section? How to design the singly reinforced rectangular section? First problem; The cross section of a rectangular beam has to resist a bending moment of 75 kNm. If the beam is 250mm wide and the permissible stresses in concrete and steel as 5 N/mm ^{2} and 190 N/mm ^{2} respectively, find the effective depth and tensile reinforcement. They have asked us to find out what is the effective depth of beam, as well as the reinforcement required for it. Now, let us begin designing. So, here this is the rectangular they have asked us to design. So, here the cross section has to resist the bending moment of M = 75 kNm and the breadth of beam is b = 250 mm and the permissive stresses in the concrete; sigmacbc = 5N/mm ^{2} and sigmast = 190 N/mm ^{2}. They have asked to find out what is the effective depth of beam (d) and what is the momentAst? So, we have two expressions i.e

moment of resistance = T x Z i.eAst sigmast (d-x/3),

this is one equation. The other equation is

C x Z = bx sigmacbc/2 (d - x/3).

The first expression is used to find out the area of steel. The second expression is used to find out the effective depth of the beam by assuming b, but b is given in the problem as 250. So, first we need to assume that the given section is a balanced section. Assume section as balanced one but to find out;

Xc. Xc/sigmacbc = d - Xc/ sigmast/m. M = 280/3sigmacbc = 18.66. Xc/ sigmacbc is 5 = d - Xc/ 190/18.66.

So we have found Xc in terms of d. Xc in terms of d is 0.89. I am going to substitute this Xc in the moment of resistance i.e

Mr = bxc sigmacbc/2 (d - Xc/3).

So, the moment is given in the problem by equating

M = Mr; 75 x 10 6 = 250 x 0.89 d x 5/2 (d - 0.89d/3),

from that I can find out what is d = 639.22 mm. This is the depth of beam, it can be equated to 640mm. Now, to find out Ast. So

M = Mr = Ast sigmast (d - xc/3).

By equating the moment of second expression and finding that is

75 x 10 6, kN = 75 x 10 6 Nmmi.eAst x 190 (640 - 0.89 x 640/3). So Ast = 693 mm^2.

Let us summarize what we have learnt so far in this lecture. In this lecture we have seen the analysis of singly reinforced rectangular beam and the design of singly reinforced rectangular beam by doing some problems. From that I hope we can understand how to find out the moment of resistance and the design of reinforced rectangular section and the Questions; So here I have given you one question to design the singly reinforced rectangular sectioni.e The cross section of an RC beam is required to resist a bending moment of 65 kNm. Find the dimensions of section and area of steel required. Assume that sigmacbc = 7 N/mm^2 and sigmast = 190 N/mm^2. In the case of references, we could refer IS code 456:2000 Plain and reinforced concrete - code of practice and one book which is written by S.N Sinha, the title being; Reinforced concrete design. With this, we come to the end of this lecture, Thank you!