Design of Structures II

Lecture 3

Welcome to UGC lecture series, in B.Architecture. Now, we are having, 'Design of Structures-II', Unit 1, lecture 3. In the lecture 2, we have seen concept of limit state method of design. What are the limit state, what are the limit states we are going to use in the case of limit state method and what are the advantages of limit state method over the other two methods. What are the difficulties we have faced in the other two methods, which have been overcome by the limit state method, so far we have discussed. Also, we have begun to design, we have started analysing the beams by means of working stress method. In the case of working stress method, in the schedule 2, we have seen before analysing the rectangular beam, we have analyzed three types of sections; one is balanced section, the other is under-reinforced section and the third one over-reinforced section. In these three sections, what are the nature of failures that occur in these three sections and how the neutral axis has moved. So, we have seen far these three type of sections in lecture 2. In lecture 3, we are going to begin analysing, the rectangular section. Based on the expression, after analysing, we are going to design the rectangular section. In the case of rectangular sections, again we have here; balanced section, under-reinforced section and over-reinforced section. How do we begin analysing? What is the aim of analysis of this rectangular section or before we look into that, what are the aim of analysis of the beams using working stress method. So, in the case of any structure, for example, if you take a beam, a beam which is simply supported by the ends, now a days subjected to some kind of forces. Due to these forces, these is a kind of momentum that develops i.e a momentum;

$m= wl^2/8$.

How to design this? How to find out the size of member and the area of reinforcement required for it? Any member which is subjected to any kind of forces, there is an internal force that is induced in these structures. Due to these forces, there is a moment which is developed due to the external force, that is called as; the external bending moment. It is called as External Bending moment, due to this there is a moment which is induced in the structure, that moment is usually called as internal moment. That internal moment which is again used to resist the external moment, this moment is called as Moment of resistance. This is the aim of analysing or analysis of the rectangular or any kind of beam. In the analysis we are going to find out what is the moment of resistance. How the moment of resistance has been used for the design of members b and d, area of reinforcement ast. Using this moment of resistance by equating this moment of resistance to the extent of moment, we are going to find out the size of section as well as the area of reinforcement. This is the aim of analysis of rectangular. This is the analysis of a general member in working stress method and here in order to find out this moment of resistance. This is a moment of resistance which is developed or induced in the structure, which is used to resist the external moment. Once, these two moments are equated, which is equated, the beam will be more safe. By equating these two moments we are going to find out the size of member as well as the area of reinforcement.

First, we are going to find out what is the moment of resistance by analysing a rectangular section. In the case of any beams, there are two types of beams; one is singly reinforced beams and the other one is; Doubly reinforced beams. What do you mean by Singly reinforced beam? The beam which is provided with a reinforcement at the tension zone is only called as Singly reinforced beam. The beam which is provided with a reinforcement both at tension as well as at the compression zone is called as Doubly reinforced beam. Under what circumstances are doubly reinforced sections designed, we will see later. Before that, we are going to see, what is singly reinforced rectangular section? How to analyze the singly reinforced sector/ section, how to find out the moment of resistance which is induced in the beam? Now, for that I am taking a section, so singly reinforced beam; the beam which is provided reinforcement at the tension joint only. When you take the cross section of the beam, this is the cross section of the beam, the reinforcement which is provided at the tension joint. So, here these are all the main reinforcements and the small d(d) is called the effective depth of the beam and b is called the width of the beam. d is effective depth of beam. What do you mean by effective depth of beam? The distance between the center of reinforcement to the topmost extreme fibre is called effective depth of beam. Overall depth of beam means, the distance between the top to bottom most fibre is called Overall depth of beam. How to find out the overall depth of beam? depth of beam + diameter of the rod/2 + clear cover. What is a clear cover of a beam? The clear cover of a beam, depends on; the environmental conditions which is given is 456, normally in our country we choose it as a moderate one. So, the clear cover of a beam is taken as 30m and the overall depth which is found from effective depth of beam + diameter of the rod/2 + clear cover, d is the effective depth of the beam which is from the topmost extreme fibre to the center of the reinforcement. In the case of working stress method, the concrete and steel are considered as an elastic material. So, the strain diagram, here it has been taken as a linear one. This is the strain diagram and if you take this as the neutral axis, this is a strain diagram. The maximum strain at the topmost extreme fibre is EPSILON C, and bottommost fibre is taken as EPSILON T. So, there is zero strain at the neutral axis. So, it has neither combustion nor tension and the stress diagrams; like here in the case of working stress method, the stress diagram is a linear one. So, here the compression portion is taken as a linear one. It varies from zero at the neutral axis to maximum at the topmost extreme fibre i.e taken as sigmacbc and this is the stress diagram. The maximum stress in the reinforcement is sigmast. This is stress diagram. In order to find out the moment of resistance, I am converting this stress diagram into an equivalent stress diagram by converting or transferring various steel into the equivalent area of the concrete surrounding tensile steel. So, this conversion is normally called as transformation of area i.e transformation of area of steel into equivalent area of concrete. That is called as transformation of area. In order to find out the depth of neutral axis, here they are all transferring entire area of steel into equal of the concrete below the neutral axis and the maximum stress at the topmost extreme fibre is taken as; sigmacbc in the concrete and what about the equivalent stress in concrete surrounding tensile steel. So, here in the case of the working stress method, I am taking the strain in steel is EPSILON T and strain in concrete as; EPSILON C. Stress in steel, we all know that sigma ST, which is found from EsEt. In the case of working stress method, from the assumption that there is a perfect bond between steel and concrete, hence no slip takes between them. So, due to that, the stress in steel, the stress in concrete, surrounding tensile steel i.e sigmact is taken as EcEt, since no slip takes place between the concrete and steel due to the perfect bond between them. Epsilon C may be taken as; which is epsilon T. Strain in concrete must be equal to strain in steel i.eEcEt, Et may be taken as sigma ST/Modulus of Steel. EPSILON T may be written as; sigma St/ Es that is equal to sigma St/ MODULUS steel/ MODULUS Concrete which is equal to sigma St/m. Where m is MODULUS of steel/ MODULUS of concrete. This is the stress in concrete surrounding tensile steel, from the transformation of area of steel into the area of concrete, either found what is stress in concrete surrounding tensile steel i.e sigma St/m. This is the stress diagram of balanced, this may be called as Equivalent stress diagram. Taken Xc the depth of neutral axis, the critical depth of neutral axis, if we take this as the balanced section and the remaining depth will be d - Xc. How do we find out the moment of resistance? The moment of resistance is equal to, what is the moment of resistance? The moment to resistance above the neutral axis, due to the compressive forces that compress the force into the distance. Another moment of resistance due to the tensile forces t into the distance. C is the compressive force, T is the tensile force, we need to find out what is the distance that is otherwise called as Lever arm. Before that, we need to find out what is the depth of neutral axis? Before, in the case of singly reinforced section, our aim is to find out the moment of resistance i.e C x Lever arm x T x lever arm, this is the moment of resistance above the neutral axis due to the total compressive force. This is the moment of resistance below the neutral axis due to tensile force. So, in the case of any beam, there are three different sections that we have discussed; one is balanced section, another one is under-reinforced section and third one is, over-reinforced section. What is the moment of resistance of the balanced section? What is the moment of resistance of the under-reinforced section? What is the moment of resistance of the over-reinforced section? So, in order to find out the moment of resistance of these three sections, we need to confirm the neutral axis i.eXc and X, we need to compare. If X is equal to Xc, it is the balanced section. If X is less than Xc, it is the under-reinforced section, if X is greater than Xc, it is over-reinforced section. In order to find out the moment of resistance of the respective section, first we need to find out the location of neutral axis i.e the actual depth of neutral axis and the location of critical depth of neutral axis. So, you assume, this is a strain diagram or an equivalent stress diagram of a balanced section. So, to find out the Xc. To find Xc from a similar triangle property, Xc/

sigmacbc at a depth of Xc the neutral axis, the stress is sigmacbc. At a depth of d - Xc from the neutral axis it is, sigma St/ m. This is the equation which is used to find out the critical depth of neutral axis. Next, we find out X i.e Actual depth of neutral axis. Actual depth of neutral axis of the unbalanced section, normally in the case of unbalanced section the neutral axis is going to be situated at the center of gravity of the section, in order to find out this x, we need to equate. By equating the moment of area on compression side to the moment of area on tensile side. In the case of unbalanced section, the neutral axis is going to be present at the center of gravity section, in order to find out that depth of neutral axis, we are going to equate the moment of area on compression side to the moment of area on the tensile side. What is the moment of area? Moment of area is area x C.G distance. What is the moment of area on the compression side? area of compression side or compression portion x distance of C.G of compression portion from the neutral axis that must be equal to, Moment of area on tensile side is equivalent to the concrete area of tensile steel x distance of C.G of tensile steel from the neutral axis. We have transferred the entire of steel into the equivalent area of the concrete. That is why we have found the stress at the bottom most extreme fibre is sigma St/m. m is the equivalent stress in concrete surrounding tensile steel. So, when you find out the moment of area of concrete, we need to consider the equivalent concrete area of the tensile steel. That has to be again multiplied by the distance of the area of tensile from the neutral axis. So, moment of area is area into distance. Now, we need to find out what is the area of the compression side. The area of the compression side is; this is compression portion. The compression portion which is normally above the neutral axis, the compression portion is b x X, this is the balanced section. I am going to find out, the depth of neutral axis for unbalanced section. You take this as the stress diagram of unbalanced section. So, here this is X and this is d - X, where X is the actual depth of neutral axis. So, here the area of compression portion is b x X, so here this is x; b x X. This is the area of the compression side and the distance of C.G of the compression side from the neutral axis is; half of this portion, that is taken as x/2 which is equal to; equivalent concrete area of the tensile steel i.e the tensile force in steel which is equal to Ast x sigmast, that we already know. Force = Stress x Area; so equivalent are; tensile force in concrete surrounding tensile steel which is equal to Asti.e when you take the tensile force as Ast x sigmast, if you take the tensile force in concrete surrounding tensile steel i.e sigmact is sigma St/m. When you take sigma St = sigmact x m. Here sigmact = sigmast/m where sigmast is considered as sigmact x m. So here, I am substituting, instead of sigma St, m x sigma Ct, if you consider stress in steel as sigma St, tensile force in steel is equal to Ast and sigma St. If you consider the stress in concrete surrounding tensile steel as sigma Ct, the equivalent tensile force surrounding tensile steel is Ast x m x sigma St. So, sigma St, has to be replaced by m x sigma Ct = mAst x sigmact = mAst is the equivalent area of concrete surrounding tensile steel. When you take sigma St, the area of steel is Ast. When you take the sigma Ct i.e the stress in concrete surrounding tensile steel, the equivalent area of concrete surrounding tensile steel is m x Ast. So, that Ast if it is multiplied the modern ratio, it will become equivalent concrete area of tensile steel surrounding it. This is called as the transformation of area from the steel into concrete.

So here equivalent concrete surrounding tensile steel is taken as; m x Ast. What is the distance of C.G tensile steel from the neutral axis? This is the tensile steel which is placed at a distance d - x, from the neutral axis. This is the equation which is used to find out the actual depth of the neutral axis i.e$bx^2/2$ = mAst x d - x, this is the formula used to find out Ast. Once you find out this one, you need to compare x with Xc and you can finalize what is the type of section we are having. If x = Xc; it is a balanced section. If x <Xci.e under-reinforced section. If x >Xc it is over-reinforced section. This is the way of finding out whether the section is balanced or unbalanced section. Once, you find out this section i.e under-reinforced or balanced section, we can easily find out what is the moment of resistance of the respective section. Now, next step is, to find out the Moment of Resistance.

So, I already told you about the neutral axis, the moment of resistance due to the compressive forces c x z and moment of resistance above the neutral axis due to tensile forces; t x z. If under-reinforced section i.e x <Xc, it is under-reinforced. What is the moment of resistance? It is T x Z. Why is it T x Z? In the case of under-reinforced section, the nature of failure is tensile failure and we are using less steel than required for the balanced section, that is why it fails due to tension. So, that is why the moment of resistance = T, the moment of distance x Z is lever arm. Lever arm means, here I am taking, C b the total compressive force which is offered by the whole concrete area above the neutral axis which is acting at the C.G of the stress diagram. If X is taken as the depth of the neutral axis, the C.G of this stress diagram from the top most extreme fibre is; X/3 and T be the total tensile force which is offered by the tensile reinforcement and the resultant distance or the distance between the resultant compressive force and the tensile is called the lever arm, Z i.e d - x/3 i.e the depth of this beam is taken as d and the x/3 is taken as the distance of the C.G of the total compressive force from the topmost extreme fibre and the remaining distance between these two forces will be called lever arm; d - x/3. So, here tensile force is normally called Ast x sigmast. Force is stress x area x (lever arm i.e d - x/3). This is the moment of resistance of the under-reinforced section. If over-reinforced section i.e x>Xc, the moment of resistance is normally C x Z. So, here the section which is normally failed due to overstress in concrete. So, the nature of failure here is; brittle failure or compression failure. The moment of resistance is C x Z. C is normally force, C is the force which is equal to stress x area. Area is = area x stress. Area here it is; area of the compression portion. Area of the compression portion is b x X. Stress is, here the stress diagram is a triangular one, it is varying from 0 to the topmost extreme fibre, so the average stress is taken as sigmacbc/2 i.ebx sigmacbc/2 (the lever arm i.e d - x/3). So, this is the way of finding the moment of resistance of the over-reinforced section. If balanced section, we can use both the expressions to find out the moment of resistance i.e x = Xci.e Mr = T x Z or C x Z by substituting x = Xci.eAst x sigmast(d - x/3). Second one is; bxc sigmacbc/2 (d - xc/3). So we can use both the expressions to find out the moment of resistance. We can get, the same answer, this is the way of finding the moment of resistance of the sections. So, here we begin to analyze the rectangular section, first we analyzed the singly reinforced beam. Singly reinforced beam meaning the beam which is provided with the reinforcement at the tension zone. The doubly reinforced beam, the beam which is provided with reinforcement at the tension zone as well as the compression zone is called as the doubly reinforced section. In the case of singly reinforced sections, we are finding the moment of resistance, why are we finding the moment of resistance is, if any force which is acting under the structure, there is an internal force which is developed due to the external force. When these two forces are equated, this beam will be safe. In order to find out the size of the section or the area of the section, I am equating the external moment to the internal moment. The external moment is moment due to the external force is called the external moment and the moment which is developed in the structure due to the external moment, to resist the external moment is called as the moment of resistance. By equating these two moments, we can easily find out the size of the section as well as area of reinforcement.

Let us have a look at the summary of Lecture 3, unit 1. In the case of working stress method, so far we have discussed the analysis of singly reinforced rectangular section and in this we have found the critical depth of neutral i.e the expression of critical depth of neutral axis and the expression for actual depth of neutral axis. Finally we have found the, moment of resistance of balanced under-reinforced and over-reinforced section, using the formula which is derived in this lecture. We are going to find out the moment of resistance, as well as we are going to design the size of section and area of reinforcement in the next lecture. Regarding lecture 3, the questions are; Determine the expression for critical depth of neutral axis, that we have seen and Determine the expression for actual depth of neutral axis and Determine the expression for Moment of resistance of under reinforced and over reinforced sections. Once you find out these three things, we can easily design the section. The references for this section are; IS 456:2000 Plain and reinforced concrete- code of practice and the book - Reinforced concrete designed by S.N Sinha. We come to the end of this session. Thank you!