Design of Structures – II

Lecture – 22

In the previous lecture we have seen how to calculate the load acting on the beam for the design of continuous beam. And we have also found the bending moment at various location using IS 456:2000 coded coefficients. Now in this lecture we are going to find out the A_{st} required for each and every moment which has been calculated using the moment coefficient which is as per IS coded coefficients. And after finding that we are going to check the depth 'd' for deflection and also we are going to draw the reinforcement detailing.

This is the table 12 given in IS 456:2000 to find out the moment at varies places for the continuous beam. In the case of continuous beam based on the nature of the beam the moment has to be found at the respective places. Before that first we have to how the beam which is deflected based on the deflection nature of the beam the moment has to be analyzed and it has to be found. Now this is the beam which is subjected to uniformly distributed load. Here the span of beam is l and due to the load the beam will be deflected like this.

This is the deflection nature of the beam and using this deflection nature this beam has to be analyzed by means of a copper analysis. That is by means of classical method of analysis that is slope distribution method, amount distribution method etc. But here in order to avoid those difficulties to analyze this continuous beam IS code has given to find out the moment at various places with certain conditions.

The first condition is the beam has minimum of three or more spans and the load must be uniform throughout the entire length of the beam. And also the span must be uniform. And if the span is different it should not varies more than 15%. Along with this condition the moment code coefficient has to be used. When you start using the moment code coefficient to find out the bending moment in the case of this beam at varies locations. The moment distribution need not be carried out. This is as for IS 456:2000.

Say in the case of continuous beam we have seen in the previous lecture we have calculated the load that is live load as well as dead load separately. Since the moment coefficient for continuous beam at varies location. And also we have found the bending moment i.e., the bending moment near the middle of end span. And we have started doing some problems to design the continuous beam. That the span of beam is 4m and the wl which is given in the problem is 25kN/m and the materials given in the problem isM20 & Fe415.

Before designing first the steps to design the size as well as the area of reinforcement load calculation has to be done. Since the moment calculation has given the dead load and live load separately the IS code moment coefficient has to be calculated separately.

i) <u>Load Calculations:</u>

First we need to calculate the dead loads self head of the beam. For finding the self head of the beam first we need to assume the beam. Normally the assumption will be based on the deflection control l/d ratio which the ratio for continuous beam is 20. So from this we can easily find what is the depth of the beam has to be 26 and depth of the beam will be l/26.

$$\frac{l}{d} = 26$$

$$d = \frac{l}{26}$$

$$= \frac{4000}{26}$$

I am assuming the height of the beam as 230 x 450mm overall and the self height of the beam that is equal to the size of beam into the respective unit weight of the section. This will be given as,

$$= 0.23 \times 0.45 \times 25$$

$$= 2.59 kN/m$$

So live load is given in this problem that is W_l which is 25kN/m.

ii) Moment Calculations:

(a) Span moments.

In the span moment we are having the moment at the middle of end span. That is given as $= \frac{1}{12} W_d l^2 + \frac{1}{10} W_l l^2$. That is the moment we have seen. By substituting the values we get,

$$= \frac{1}{12} \times 2.59 \times 4^2 + \frac{1}{10} \times 25 \times 4^2$$

$$= 43.451 kNm.$$

The next moment is moment at the middle of interior span. That is equal to,

$$= \frac{1}{16} W_d l^2 + \frac{1}{12} W_l l^2$$

$$= \frac{1}{16} \times 2.59 \times 4^2 + \frac{1}{12} \times 25 \times 4^2$$

$$= 35.92 kNm.$$

b) Support Moments.

This is moment at the support next to end support. That is given by,

$$= -\frac{1}{10} W_d l^2 - \frac{1}{9} W_l l^2$$

$$= -\frac{1}{10} \times 2.59 \times 4^2 - \frac{1}{9} \times 25 \times 4^2$$

$$= -48.58 kNm.$$

Next we need to find moment at other interior supports. That is given by,

$$= -\frac{1}{12} W_d l^2 - \frac{1}{9} W_l l^2$$

$$= -\frac{1}{12} \times 2.59 \times 4^2 - \frac{1}{9} \times 25 \times 4^2$$

$$= -47.89 kNm.$$

Normally in the case of among four span the maximum bending moment must normally occurs at the support next to the end support. But in the case of beam the depth of the beam has to be designed against the maximum bending moment. In case of finding the area of reinforcement the area of reinforcement has to be separately found from all bending moments. Now this value should be selected to find the effective depth of the beam.

iii) Effective depth of beam:

We have assumed the depth of the beam as 230 x 450mm. So by equating we get,

$$M_{u\max} = M_{u\lim}$$

$$M_{u\max} = 1.5 \times 48.58 = 72.87 kNm.$$

$$M_{u\lim} = 0.138 f_{ck} bd^3 = M_{u\max}$$

$$d = \sqrt{\frac{M_{u\max}}{0.138 f_{ck} b}}$$

$$= \sqrt{\frac{72.87 \times 10^6}{0.138 \times 20 \times 230}}$$

$$= 338.80 mm.$$

So overall depth of the beam D is equal to (338.80+30+20) 378.80 which is less than 450mm. Hence the beam is safe against flexure. This effective depth of the beam has to be again checked against the limit state of serviceability i.e., the deflection criteria. That is has to be checked at the end of the beam.

iv) To find A_{st}:

Now we have kept D is equal to 450mm and d is equal to (450-30-20/2) 410mm for the first bending moment,

$$M_u = 0.87 f_y A_{st} (d = 0.416 x_u)$$

$$M_u = 43.45 \times 1.5 = 65.175 kNm$$

So x_u here it is,

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b}$$

$$x_u = \frac{0.87 \times 415 \times A_{st}}{0.36 \times 20 \times 230}$$

$$= 0.218 A_{st}$$

$$65.175 \times 10^6 = 0.87 \times 415 \times A_{st}(410 - 0.416 \times 0.218 x_u)$$

$$A_{St}^2 - 4521.40 A_{st} + 1.99 \times 10^6 = 0$$

By solving this quadratic equation we will get two values which we should substitute to find bending moment value. Since the beam is designed as under reinforced or singly reinforced.

$$A_{St_{(req)}} = 494.13 mm^2$$

And also I am providing the 16mm bar that the number of bars is equal to,

$$= \frac{494.13}{\pi/4 \times 16^2}$$

$$= 2.546 \cong 3 nos.$$

Provided 3 number of 16mm bar for resisting the bending moment near the middle of end span.

Next A_{St} required at the middle of other interior span. So here the bending moment is,

$$M_u = 1.5 \times 35.92 = 53.88 kNm$$

$$M_u = 0.87 f_y A_{st}(d = 0.416 x_u); x_u = 0.218 A_{st}$$

$$53.88 \times 10^6 = 0.87 \times 415 \times A_{st}(10 - 0.416 \times 0.218 A_{st})$$

$$A_{St}^2 - 4521.40A_{St} + 1.65 \times 10^6 = 0$$

$$A_{St(req)} = 400.39mm^2$$

Here I am using 16mm diameter bar, number of bars required for this A_{St} is 2 x 16mm.

In a similar way we are going to find the $A_{St(req)}$ for resisting moments at the support next to end support.

$$M_u = 1.5 \times 48.58 = 72.87kNm\text{s}$$

$$M_u = 0.87 f_y A_{st}(d = 0.416x_u); x_u = 0.218A_{st}$$

$$72.37 \times 10^6 = 0.87 \times 415 \times A_{st}(410 - 0.416 \times 0.218A_{st})$$

$$A_{St(req)} = 563.4mm^2$$

Here by using the 16mm diameter the number of bars required will be 3 numbers.

The A_{st} required for resisting the moment at other interior support. So the moment we have found as,

$$M_u = 1.5 \times 47.89 = 71.84kNm$$

$$M_u = 0.87 f_y A_{st}(d = 0.416x_u); x_u = 0.218A_{st}$$

$$71.84 \times 10^6 = 0.87 \times 415 \times A_{st}(410 - 0.416x_u)$$

$$A_{St}{}^2 - 4521.40A_{st} + 2.19 \times 10^6 = 0$$

$$A_{St(req)} = 351.68mm^2$$

Again here I am using 16mm diameter bar, number of bars needed to resist this bending moment is 3 numbers of 16mm diameter bar. This is the way of finding the area of reinforcement for resisting four moment at the respective places. Now before seeing the arrangement or detailing of the reinforcement at the various places we need to check the depth of the beam making the limit state of the serviceability. So here the case of maximum bending moment A_{st} required is 563.4mm^2 and we have provided three numbers of 16mm bar. Here Ast provided for the maximum bending moment is 603.19mm^2. Here we are checking the depth of beam against the deflection.

So the l/d ratio for the continuous beam is 26. And depending upon the type and stress in the reinforcement this factor I did'nt modified by multiplying with the modification factor. This M.F has to be found from the graph of figure 4 of IS 456:2000. That is the figure shows the modification factor verses percentage reinforcement.

For finding the modification factor first we need to find out the percentage of reinforcement as,

$$p_t = \frac{100 A_{st(req)}}{ld}$$

$$= \frac{100 \times 608.19}{230 \times 410}$$

$$= 224.83 N/mm^2$$

$$f_s = 0.58 f_y \frac{A_{st(req)}}{A_{st(req)}}$$

$$= 0.58 \times 415 \times \frac{563.42}{603.19}$$

$$= 224.83 N / mm^2$$

From the deflection criteria l/d ratio is equal to 26 into modification. So d here required is equal to,

$$d = \frac{l}{26 \times M.F}$$

$$= \frac{4000}{26 \times 1.2} = 128.21 mm$$

This is less than 410mm. Hence it is safe. For the moment the depth is 378.80 and the provided depth is 410 mm. So for the deflection criteria the depth needed is 128.21mm and the provided is 410mm. The design of reinforcement is calculated. Here we have the four spans which is continuous beam and the moment present in the middle of span is 3 numbers of 16mm diameters. And for the support it needs three numbers of 16mm diameters. It has been provided for a distance of 0.3l on either side of the beam. There is a remaining reinforcement which is extended throughout the entire length of the beam. To provide or to hold the shearing force of 2 x 16 mm and other interior also we have found the minimum reinforcement required for these three numbers.

Summary:

In this lecture we have studied the reinforcement required for the continuous beam at varies places. And also we have seen the depth of the beam again deflection and the reinforcement of the continuous beam which has to be provided at varies place. This is as for IS 456:2000.

Questions:

1. Design beam and the area of reinforcement required for a five span continuous beam of effective span 5m subjected to a total live load of 23 kN/m. Use M25 and Fe500 as materials.

Reference:

- IS 456:2000 plain and reinforced concrete – code of practice
- Reinforced concrete design by S.N. Sinha