Design of Structures– II

Lecture – 19

In the previous lecture we have seen in detail about the moment of resistance of the flanged section that is singly reinforced flanged section. And we get practiced to do certain examples in finding the moment of resistance of the singly reinforced section. Now in this lecture we are going to have a detailed analysis of doubly reinforce flanged sections for three different cases.

Analysis of doubly reinforced flanged sections to determine the M_u for the following cases,

i. When $X_u \le D_f$

ii. When $X_u > D_f$ & $0.43X_u \ge D_f$

iii. When $X_u > D_f$ & $0.43X_u \le D_f$

Case (i): When $X_u \le D_f$

When the neutral axis lies within the flanged section or just falls at the bottom of the flange. This is the T-beam we need to provide the reinforced section at the top as well as at the bottom that's why it is called doubly reinforced section. The tensile strength d is placed from the top. Here ε_{cu} is the maximum strain at the top most extreme fiber that is 0.0035. Here I am taking x as the depth of the neutralized axis and C_u be the compressive force offered by the whole concrete area above the neutral axis.

And $0.446 f_{ck}$ is the maximum stress at the top most extreme fiber. Since the steel is placed at the distance d’ from the top most extreme fiber here is another force C_{u2} which is due to the compressive reinforcement which is placed at the compression zone at a distance d’ from the top. The C_{u1} is the total compressive force offered by the whole concrete area above the neutral axis which is acting at $0.416X_u$ from the top most extreme fiber and this is the Cg of total compressive force.

The distance between T_{u1} the corresponding tensile force and C_{u1} is the lever on z_1 that is $(d - 0.416X_u)$. And T_{u2} be the additional tensile force offered by the force which is used to balance the C_{u2}. So C_{u2} which is acting at a distance from the top most extreme fiber and the resultant between the tensile force T_{u2} and the resultant compressive force C_{u2} is the lever on z_2 that is $(d - d')$.

To find out X_u at the neutral axis,

$$C_u = T_u$$

$$C_{u1} + C_{u2} = T_u$$

$$C_{u1} + C_{u2} = 0.87 f_y Ast$$

Here A_{st} is A_{st1} that is used to T_{u1} and A_{st2} is additional tensile reinforcement two balanced T_{u2}. The total A_{st} is A_{st}. Here force is normally stress into area and $C_{u2} = A_{st}(f_{sc} - f_{cc})$.

$$0.36 f_{ck} x_u b_f + A_{st}(f_{sc} - f_{cc}) = 0.87 f_y A_{st}$$

This is the expression which is used to find out the depth of neutral axis. Here we need to find out the corresponding moment of resistance. Since the neutral axis lies within the flange it is rectangular section.

$$X_u < X_{u\,\mathrm{lim}}$$

When this condition satisfies then it is under reinforcement action and if

$$X_u > X_{u\,\mathrm{lim}}$$

Then it is designed as doubly reinforced section. Here the corresponding moment of resistance will be,

$$M_u = C_{u1} \times z_1 + C_{u2} \times z_2$$

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$$= 0.36 f_{ck} x_u b_f (d - 0.416 x_u) + A_{sc}(f_{sc} - f_{cc})(d - d')$$

$$M_u = T_{u1} \times z_1 + T_{u2} \times z_2$$

$$= 0.87 f_y A_{st1}(d - 0.416 x_u) + 0.87 f_y A_{st2}(d - d')$$

This is the moment of resistance for the first case.

Case (ii): When $X_u > D_f \ \& \ 0.43X_u \geq D_f$

Here it has b_f and D_f and has the area d' from the top. It has also $A_{sc} \ \& \ A_{st}$ and b_w. Now the neutral axis lies outside the flange.

$$= \frac{f_y}{1.15E_s} + 0.002$$

The beam has two portions one is the central rectangular portion and another one is the remaining flange portion that is $b_w \times d \ \& \ (b_f - b_w)D_f$. Here since the steel is placed in the distance d' from the top most extreme of the fiber. In this I am taking C_{u1} is the total compressive force and which is acting at $0.416X_u$ from the top and T_{uw} is the corresponding tensile force and the distance between resultant force is which is $z_1(d - 0.416x_n)$. And in addition to that we have placed the steel at a distance d' from the top most extreme fiber. Again here C_{uf} is the compressive force offered by the flange portion. Throughout the flange portion the stress is uniform.

Now to find X_u at the neutal axis,

$$C_u = T_u$$

$$C_{uw} + C_{u1} + C_{uf} = T_u$$

$$0.36 f_{ck} x_u b_w + A_{sc}(f_{sc} - f_{cc}) + 0.446 f_{ck}(b_f - b_w)D_f$$

$$= 0.87 f_y A_{st}$$

So this is the equation used to derive or to find out the depth of neutral axis. And C_{uw} is the compressive force offered by the web portion and C_{uf} is the compressive force offered by the flange portion. These two are the total compressive force offered by the whole concrete above the neutral axis. Now we need to find out the moment of resistance.

$$M_u = C_{uw} \times z_1 + C_{u1} \times z_2 + C_{uf} \times z_3$$

$$= 0.36 f_{ck} x_u b_w (d - 0.416 x_u) + A_{sc}(f_{sc} - f_{cc})(d - d') + 0.446 f_{ck}(b_f - b_w) D_f (d - D_f / 2)$$

$$M_u = T_{uw} \times z_1 + T_{uz} \times z_2 + T_{uf} \times z_3$$

$$= 0.87 f_y A_{stw} (d - 0.416 x_u) + 0.87 f_y A_{st} (d - d') + 0.87 f_y A_{stf} (d - D_f / 2)$$

This is the moment of resistance for the corresponding case.

Case (iii): When $X_u > D_f \ \& \ 0.43 X_u \le D_f$

Taking a doubly reinforced beam to derive a moment of resistance for the third case here b_f is the width of flange, D_f is the depth of flange, d is the effective depth of the section from the top to the center of tensile reinforcement. And A_{sc} is the area of compressive reinforcement which is placed at d' from the top most extreme fiber. Here the neutral axis lies outside the flange. The tensile strength is,

$$= \frac{f_y}{1.15 E_s} + 0.002$$

In this case it has two different portions, one is rectangular portion and another one is a small parabolic portion.

$$y_f = 0.15x_u + 0.65D_f < D_f$$

Here I am dividing the portion into two as central rectangular portion and another one as the remaining flange portion that is $b_w \times d \;\&\; b_w \times y_f$. Then C_{uw} is the compressive force offered by the web portion normally acting at $0.416x_u$ and T_{uw} is the corresponding tensile force which is equal to $z_1(d - 0.0416x_n)$. And again another C_{u1} is the compressive force offered by the reinforcement which is called as d' from the top most extreme fiber and $z_2(d - 0.0416x_n)$.

Now to find out X_n, at the neutral axis,

$$C_u = T_u$$

$$C_{uw} + C_{u1} + C_{ut} = T_u$$

$$= 0.36 f_{ck} x_u b_w + A_{sc}(f_{sc} - f_{cc}) + 0.446 f_{ck}(b_f - b_w) y_f$$

$$= 0.87 f_y A_{st}$$

This is the expression we need to calculate the depth of the neutral axis. Now we need to find out the moment of resistance,

$$M_u = C_{uw} \times z_1 + C_{u1} \times z_2 + C_{uf} \times z_3$$

$$= 0.36 f_{ck} x_n b_w (d - 0.416x_n) + A_{st}(f_{sc} - f_{cc})(d - d')$$
$$+ 0.446 f_{ck}(b_f - b_w) y_f (d - y_f / 2)$$

Where $y_f = 0.15x_n + 0.65D_f < D_f$

$$M_u = T_{uw} \times z_1 + T_{u1} \times z_2 + T_{uf} \times z_3$$

$$= 0.87 f_y A_{stw}(d - 0.416x_n) + 0.87 f_y A_{st1}(d - d')$$
$$+ 0.87 f_y A_{stf}(d - y_f / 2)$$

This is the required moment of resistance for the corresponding case.

Summary:

In this lecture we analysed the doubly reinforced flanged section of for finding the moment of resistance for the following cases,

i. When $X_u \le D_f$

ii. When $X_u > D_f$ & $0.43X_u \ge D_f$

iii. When $X_u > D_f$ & $0.43X_u \le D_f$

Questions:

1. Find the M_u for doubly reinforced flanged section for the following cases

   i. When $X_u \le D_f$

   ii. When $X_u > D_f$ & $0.43X_u \ge D_f$

   iii. When $X_u > D_f$ & $0.43X_u \le D_f$

Reference:

- IS 456:2000 plain and reinforced concrete – Code of practice
- Reinforced concrete design by S.N Sinha.