Design of Structures – II

Lecture-18

Here we are having a look on the subject the design of structure II. In the previous lecture we have seen how to calculate the moment of resistance of the singly reinforced section, we have the three different cases and we have analyzed and found moment of resistance for the three cases. In this Lecture we are going to do some examples on the three cases.

Limit State Design method:

Analysis of singly reinforced flanged section to determine the M_u on worked examples for the following cases that is for,

i. When $X_u \leq D_f$

ii. When $X_u > D_f \ \& 0.43X_u \geq D_f$

So we need to work out some problems on these three cases. The first problem is,

1. Find out the ultimate moment of resistance of a T-beam with the following data. Width of flange = 1100mm; Depth of flange = 120mm; width of rib = 275mm; Effective depth = 450mm; Ast = 2700mm^2; Materials = M25 and Fe500.

Solution:

Here they given the dimension of the problem is they given the singly reinforced section. Here the data give are $b_f = 1100mm$; and the $D_f = 120mm$; the width of rib is 275mm, effective depth d is 450mm, Ast is 2700mm^2. Materials given are M25 and Fe500. They have ask as to find out what is the moment of resistance.

In this T-beam initially we don't know whether the neutral axis lies within the flange or outside the flange. First we need to assume the neutral axis lies within the flange. We need to use the respective expression or equation to

find the depth of the axis. Then we compare the depth of the neutral axis, if the $X_u \lhd D_f$ out assumption is correct and we can use the respective formula to find out the moment of resistance. If $X_u \rhd D_f$ then our assumption is wrong so we move on to second case. Then we need to find out again the depth of neutral axis in the corresponding expression. In this case if $X_u \rhd D_f$ and we need to check $0.43X_u \geq D_f$ and if not the assumption is wrong and then we move on to the third case $X_u \rhd D_f \,\&\, 0.43X_u \leq D_f$ we need to again find out new expression for fining out X_u. By using this we need to find out whether the neutral axis lies within the flange that is $X_u \rhd D_f \,\&\, 0.43X_u \leq D_f$ and this condition satisfies then this confirms it is case 3. Then we can use the expression of respective case and find out the moment of resistance.

In any problem we need to write first the data given in the problem at the beginning.

$b_f = 1100mm, D_f = 120mm, d = 450mm, b_w = 275mm, Ast = 2700mm^2,$
$f_{ck} = 25N/mm^2, f_y = 500N/mm^2$

Here we are asked to find out the Moment of resistance for neutral axis lies within the flange that is case 1.

Case (i): $x \leq D_f$

Now to find out x at the neutral axis $C_u = T_u$,

$$0.36 f_{ck} x_u b_f = 0.87 f_y A_{st}$$

$$X_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 500 \times 2700}{0.36 \times 25 \times 1100}$$

$$= 118.636mm$$

We need to compare this X_u with D_f , $x \le D_f$ and it is Our assumption is correct. Then moment of resistance,

$$X_{u\lim} = 0.456d = 0.456 \times 450 = 205.2mm$$

$$M_u = T_u \times Z$$

$$= 0.87 f_y Ast(d - 0.416x_u)$$

$$= 470.56kNm.$$

This is the way of analyzing the T section of neutral axis lies within the flange. Now we will move on to the second problem.

2. Compute the ultimate moment of resistance of a T-beam with the following data. Width of flange = 1500mm; depth of flange = 100mm; width of rib = 300mm; effective depth = 600mm; Ast = 4500mm^2; Materials = M20 and Fe415.

Solution:

Here the given data are Width of flange = 1500mm; depth of flange = 100mm; width of rib = 300mm; effective depth = 600mm; Ast = 4500mm^2; Materials = M20 and Fe415. Now to find X_u assume neutral axis lies within the flange that is $X_u \le D_f$. At the neutral axis I am using the respective expression to find the depth of the neutral axis.

$$C_u = T_u$$

$$0.36 f_{ck} x_u b_f = 0.87 f_y Ast$$

$$X_u = \frac{0.87 f_y Ast}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 415 \times 4500}{0.36 \times 20 \times 1500}$$

$$= 150.44mm$$

Now comparing this X_u value with the value of D_f, here $X_u \triangleright D_f$. So our assumption is wrong. We will move on to the second case that is $X_u \triangleright D_f ; 0.43X_u \geq D_f$. And here I am using the expression to find the value of X_u.

$$C_u = T_u$$

$$0.36 f_{ck} x_u b_w$$

Here C_{uw} is the compressive force offered by the central rectangular portion.

$$0.36 f_{ck} x_u b_w + 0.446 f_{ck} (b_f . b_w) D_f = 0.87 f_y Ast$$

$$0.36 \times 20 \times X_u \times 300 + 0.446 \times 20 \times (1500 - 300)100 = 0.87 \times 415 \times 4500$$

$$X_u = 256.63mm > D_f .$$

At the same time we need to confirm whether $0.43X_u \geq D_f$ if it satisfies we can confirm it is case two.

$$0.43X_u = 0.43 \times 256.63$$

$$= 116.94mm > D_f$$

Hence it is coming under case 2. And we are going to use the respective expression to find the moment of resistance.

$$M_u = C_{uw} \times z_1 + C_{uw} \times z_2$$

$$= 0.36 f_{ck} X_u b_w (d - 0.416 X_u) + 0.446 f_{ck} (b_f - b_w) D_f (d - D_f / 2)$$

$$= 0.36 \times 20 \times 256.63 \times 300(600 - 0.416 \times 256.63)$$
$$+ 0.446 \times 20(1500 - 300)100(600 - 100/2)$$

$$= 862.13 kNm$$

This is the moment of resistance and this is the way to find out the moment of resistance for the second case.

3. Compute the ultimate moment of resistance of a T-beam with the following data. Width of flange = 1250mm; Depth of flange = 100mm; Width of rib = 250mm; Effective depth =650mm; Ast = 2800mm^2; materials = M20 and Fe415.

Solution:

In the third problem they have given again the T-section that is singly reinforced section and they were asked as to find out the moment of resistance.

The given data here are Width of flange = 1250mm; Depth of flange = 100mm; Width of rib = 250mm; Effective depth =650mm; Ast = 2800mm^2; materials = M20 and Fe415. First assume the neutral axis lies within the flange that is case 1.

Case (i): When $X_u \le D_f$

$$C_u = T_u$$

$$0.36 f_{ck} X_u b_f = 0.87 f_y Ast$$

$$X_u = \frac{0.87 f_y Ast}{0.36 f_{ck} b_f}$$

$$= \frac{0.87 \times 415 \times 2800}{0.36 \times 20 \times 1250}$$

$$= 112.33mm > D_f$$

Hence our assumption is wrong. So it is coming under case 2 which is $X_u > D_f$ **.**

Case (ii): When $X_u > D_f$ **;** $0.43X_u \ge D_f$

At the neutral axis,

$$C_u = T_u$$

$$C_{uw} + C_{uf} = T_u$$

$$0.36 f_{ck} X_u b_w + 0.446 f_{ck} (b_f - b_w) D_f = 0.87 f_y Ast$$

$$0.36 \times 20 \times X_u \times 250 + 0.446 \times 20(1250 - 250)100 = 0.87 \times 415 \times 2800$$

$$X_u = 66.07mm < D_f$$

Here the case is not satisfied but in the previous case it says that the case two will be satisfied. So let us try for the third case.

Case (iii): When $X_u > D_f ; 0.43X_u < D_f$

At the neutral axis,

$$C_u = T_u$$

$$C_{uw} + C_{uf} = T_u$$

$$0.36 f_{ck} X_u b_w + 0.446 f_{ck} (b_f - b_w) y_f = 0.87 f_y Ast$$

$$y_f = 0.15X_u + 0.65D_f = 0.85X_u + 0.65 \times 100$$

$$y_f = 0.65X_u + 65$$

$$0.36 \times 20 \times X_u \times 250 + 0.446 \times 20(1250 - 250)(0.65X_u + 65)$$
$$= 0.87 \times 415 \times 2800$$

$$X_u = 104.19mm > D_f$$

$$0.43X_u = 0.43 \times 104.19 = 44.80mm < D_f$$

So it is confirm that it is coming under case 3. Then to find the moment of resistance,

$$M_u = C_{uw} \times z_1 + C_{uf} \times z_2$$

$$= 0.36 f_{ck} X_u b_w (d - 0.416X_u) + 0.446 f_{ck} (b_f - b_w) y_f (d - y_{fb})$$

Where $y_f = 0.15X_u + 0.65D_f = 0.15 \times 104.19 + 0.65 \times 100$

$$M_u = 552.27kNm$$

Summary:

Let us summarize this lecture, in this session we have analyzed singly reinforced rectangular beam. We have worked on three examples for three cases and we have found the moment of resistance for the three cases.

Analysis of singly reinforced flanged section to determine the M_u on worked examples for the following cases that is for,

i. When $X_u \leq D_f$

ii. When $X_u > D_f \ \& 0.43X_u \geq D_f$

iii. When $X_u > D_f \ \& 0.43X_u \leq D_f$

Questions:

1. Compute the ultimate moment of resistance of a T-beam with the following data. Width of flange = 1450mm; depth of flange = 120mm; width of rib = 300mm; effective depth = 525mm; Ast = 3700mm^2; Materials = M25 and Fe500.
2. Compute the ultimate moment of resistance of a T-beam with the following data. Width of flange = 1500mm; depth of flange = 100mm; width of rib = 300mm; effective depth = 600mm; Ast = 4500mm^2; Materials = M20 and Fe415.