Design of Structures – II

Lecture-17

Here we are having a look on the subject Design of structures II, in the previous lecture we have seen how to calculate the moment of resistance of doubly reinforced section and how we can design the doubly reinforced section. In this lecture we are moving on to how to find out moment of resistance of the flanged sections. The content of this lecture is analysis of singly reinforced flanged sections in detail under limit state design method. First we will see the singly reinforced section. In the case of flanged section as we have discussed in the working stress method here also we are having two different cases. One is when the neutral axis lies within the flange or just lies on the bottom of the flange i.e., when $X_u \leq D_f$. The second case when $X_u \rhd D_f$ and $0.43X_u \geq D_f$ that is when the neutral axis lies outside the flange what will be the moment, in the second case we have further two cases. Next when $X_u \rhd D_f$ and $0.43X_u \leq D_f$

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Analysis of Singly reinforced Flanged sections:

When X_u is greater than or equal to D_f, here we need to find out the moment of resistance. The moment of resistance is normally equal to moment of resistance offered by the compressive force and the moment of resistance offered by tensile force. Force is normally stress into area, so in order to find out the C_u and we need to first find out what is the depth of neutral axis for the section. Whether find out the depth of the neutral axis we are equating the total compressive force to the total tensile force we can find out what will be the X_u.

And by using X_u we are going to find the moment of resistance of the sections. Here this is the T section the first section is $X_u \leq D_f$. Taking b_f which is the breadth of the flange and D_f is the depth of the flange. This is the area of the tensile strength. The first case the neutral axis lies in the flange and in the bottom of the flange. This is the stress diagram with tensile force, tensile compressive force offered by the whole concrete area.

Since the neutral axis lies within the flange, the area of the within the flange is b_f .x . First we need to find out the neutral axis that is depth of neutral axis. At the neutral axis $C_u=T_u$, here C_u is normally total compressive force of whole concrete above the neutral axis which we already derived as $0.36 f_{ck} x_u b$. here instead of b we need to substitute b_f.

$$0.36 f_{ck} x_u b_f = 0.87 f_y A_{st}$$

$$x_u = \frac{0.87 f_y A_{st}}{0.36 f_{ck} b_f}$$

We all know that $X_{u\ lim}$, so here this is something like a rectangular section. So when the neutral axis lies within the flange the case of the $T_V$the beam has to be analyzed as a rectangular section and designed as a rectangular section of size b_f/d. Now here we have already found the $x_{u\ lim}$ for varies grades of strain.

That is $x_{u\ lim}$ = 0.531d for mile steel and 0.479d for Fe415 and 0.456d for Fe500. Now we compare if X_u is less than or equal to d_f neutral axis lies within the flange. Since here the neutral axis lies within the flange we have confirmed the section has to be analyzed as a rectangular section of size b_f/d. So the moment of resistance that is equal to the tensile force is equal to T_u.Z that is 0.87 f_y A_z. Here the total beneficial is compressive force which is normally acting that is 0.416 x_u from the top most section.

So here it is the distance between two forces $(d - 0.416x_u)$ that is called lever on. If X_u is less than the $X_{u\ lim}$ than it is the moment of resistance of a given section M_u.

$$M_u = T_u \times Z$$

$$= 0.87 f_y A_{st} (d - 0.416x_u)$$

$$M_u = C_u \times Z$$

$$= 0.36 f_{ck} x_u b_f (d - 0.416 x_u)$$

This is the moment of resistance of the section which is offered by the compressive force. So we can use these two equations to find the moment of resistance of the T section when the neutral axis lies within the flange.

Case (ii):

When $x_u \triangleright D_f \ \& 0.43 x_u \geq D_f$,and if the neutral axis lies outside the flange. What would be the moment of resistance of the section?

a. Compressive force $C_{uw} = 0.36 f_{ck} x_u b_w$
b. Compressive force $C_{uf} = 0.446 f_{ck} D_f (b_f - b_w)$
c. Total tensile force $T_u = 0.87 f_y A_{st}$
d. To find Neutral Axis x_u

$$@ N.A, C_u = T_u ; C_{uw} + C_{uf} = T_u$$
$$0.36 f_{ck} x_u b_w + 0.446 f_{ck} D_f (b_f - b_w) = 0.87 f_y A_{st}$$

Here the neutral axis lies outside the flange the portion above the neutral axis is in the shape of T section that why we had did this into two sections one as the rectangular portion and another one is flange portion. Then what is C_u , it consist of C_{uw}+C_{uf} = T. What is the moment of resistance due to the portions. Normally here in the case 0.43 is greater than D_fSo this is the uniform depth of stress. The depth of stress from the top is uniform since the point 0.43x_u is greather than D_f.

$$0.36 f_{ck} x_u b_w + 0.446 f_{ck} D_f (b_f - b_w) = 0.87 f_y A_{st}$$

This is the formula to find out X_u for T section. Now we need to find out moment of resistance. That is,

$$M_u = C_u \times z$$

Here C_u consist of,

$$= C_{uw} \times z_1 + C_{uf} \times z_2$$

So when you take the rectangular portion here, C_{uw} is the compressive force offered by the central rectangular portion which is normally acting at $0.416X_u$ from the top. And the corresponding tensile force is T_{uw} and the distance between this two force is $z_1(d - 0.416x_u)$. In the case of flange portion the stress is uniform throughout the entire portion. And the total compressive force acting here is the flange portion $\frac{D_f}{2}$ from the top. And T_{uf} be the corresponding tensile force. The distance between two portion will be $z_2(d - \frac{D_f}{2})$.

$$= C_{uw} \times z_1 + C_{uf} \times z_2$$

$$= 0.36 f_{ck} x_u b_w (d - 0.416 x_u) + 0.446 f_k (b_f . b_w) D_f (d - \frac{D_f}{2})$$

This is the moment of resistance of the compressive force and the moment of resistance of tensile force is,

$$M_u = T_{uw} \times z_1 + T_{uf} \times z_2$$

$$= 0.87 f_y A_{stw} \times (d - 0.416 x_u) + 0.87 f_y A_{stf} (d - D_f / 2)$$

This is the moment of resistance for the case 2. Now we will move on to case 3.

Case (iii):

When $x_u \triangleright D_f \ \& 0.43x_u \geq D_f$

1. Ultimate moment of resistance (M_u)

$$M_u = C_{uw} \times z_1 + C_{uf} \times z_2$$

$$= 0.36 f_{ck} x_u b_w (d - 0.416x_u) + 0.446 f_{ck} Y_f (b_f - b_w)(d - Y_f / 2)$$

$$M_u = T_{uw} \times z_1 + T_{uf} \times z_1$$

$$= 0.87 f_y A_{stw} (d - 0.416x_u) + 0.87 f_y A_{stf} (d - Y_f / 2)$$

Here the neutral axis lies outside the flange the beam has to be analyzed and design as a T beam. I am dividing these T beam into two portions one is central rectangular portion and another one is the remaining flange portion. C_{uw} be the total compressive force offered by the central rectangular portion which is normally acting as $0.416x_u$ and the corresponding tensile force is T_{uw}. The distance between these two forces is $z_1(d - 0.416x_u)$. And C_{uf} is the compressive force offered by the flange portion. This must be acting at the C_g of the flange portion.

$$y_f = 0.15x_u + 0.65D_f$$

To find x_u,

$$C_{uw} + C_{uf} = T_u$$

$$= 0.36 f_{ck} x_u b_w + 0.446 f_{ck} (b_f - b_w) y_f = 0.87 f_y A_{st}$$

$$y_f = 0.15x_u + 0.65D_f \triangleleft D_f$$

To the moment of resistance the first expression is,

$$M_u = C_u \times z$$

$$M_u = C_{uw} \times z_1 + C_{uf} \times z_2$$

$$= 0.36 f_{ck} x_u b_w (d - 0.416 x_u) + 0.446 f_{ck} Y_f (b_f - b_w)(d - Y_f / 2)$$

$$M_u = T_{uw} \times z_1 + T_{uf} \times z_1$$

$$= 0.87 f_y A_{stw} (d - 0.416 x_u) + 0.87 f_y A_{stf} (d - Y_f / 2)$$

So we have found the moment of resistance of the T section for the two different cases when neutral axis lies within the flange and neutral axis lies outside the flange.

Summary:

In this lecture we have seen analysis of singly reinforced flanged section. And we have analyzed and found the moment of resistance for three cases where the neutral axis outside the flange, when the neutral axis found within the flange.

Questions:

1. Find the M_u for Singly reinforced Flanged section for the following cases.
   i. When $X_u \le D_f$
   ii. When $X_u \le D_f \ \& \ 0.43 x_u \ge D_f$
   iii. When $X_u \triangleright D_f \ \& \ 0.43 x_u \le D_f$

Reference:

- IS 456:2000 plain and reinforced concrete – code of practice
- Reinforced concrete design by S.N. Sinha