Design of Structures II

Lecture 16

Today we are going to have a look on the subject Design of structures II, in the previous lecture we have seen how to find out the moment of resistance of doubly reinforced rectangular section and a limit state design method. In this lecture we are going to see some problems on finding the ultimate moment of resistance of doubly reinforced rectangular section. And also we are going to do some problems on design of doubly reinforced rectangular sections and the design we are going to find out be the A_{st1}, A_{st2}, and $A_{sc.}$

Analysis of doubly reinforced rectangular sections:

1. In this problem we are going to find the moment of resistance of a given section,

a) An RC beam of 300mm x 500mm effective is reinforced with 804mm^2 and 1520mm^2 of steel at the top and bottom respectively. Find the M_U the beam using M20 and Fe415 as materials. d'=33mm.

Solution:

Here they are asking to find out the doubly reinforced rectangular section of b = 300mm and d = 500mm, A_{st} which is provided at the bottom is 1520mm^2 and A_{sc} which is at the top is d' = 33mm and the value of A_{sc} is 804mm^2.

Given:

b=300mm; d=500mm; d'=33mm; A_{st}=1520mm^2;

A_{sc}=804mm^2; materials = M20 & Fe415; M_U=?

The moment of doubly reinforced rectangular section M_u is equal to the moment of resistance of balanced reinforced section plus the moment of resistance due to the compressed reinforcement.

$$M_u = 0.36 f_{ck} x_{u\lim} b(d - 0.416 x_{u\lim}) + A_{sc}(f_{sc} - f_{cc})(d - d')$$

So from this we need to find out what is $x_{u\lim}$, $x_{u\lim}$ is given in the grade of steel is M20 and Fe415.

$$x_{u\lim} = 0.479d$$

$$= 0.479 \times 500$$

$$= 239.5mm$$

Next we need to find out f_{sc}, the stress in steel. Now we are going to find out the strain at d'. I'm going to find the stress at the depth of $(x_c - d')$ neutral axis.

$$\frac{0.0035}{x_{u\lim}} = \frac{?}{(x_{u\lim} - d')}$$

$$Strain? = \frac{0.0035}{x_{u\lim}}(x_{u\lim} - d')$$

$$= \frac{0.0035}{239.5}(234.5 - 33)$$

$$= 0.00302$$

$$Stress f_{sc} = 354.08N/mm^2$$

So from the tabular column given the grade steels at various stress levels, the stress is 360.9 for the strain of 0.00380.

$$0.00276 = 351.8$$

$$0.00380 = 360.9$$

Here we need to find out the stress for the corresponding strain of 0.00302. To find f_{cc}, the f_{cc} at d' is equal to 0.446 f_{ck}.

$$= 0.446 \times 20$$

$$= 8.92 N / mm^2$$

By using the expression moment of resistance M_u is equal to the moment of resistance of balanced reinforced section plus the moment of resistance due to the compressed reinforcement.

$$M_u = 0.36 f_{ck} x_{u\lim} b(d - 0.416 x_{u\lim}) + A_{sc}(f_{sc} - f_{cc})(d - d')$$

$$= 0.36 \times 20 \times 239.5 \times 300(500 - 0.416 \times 239.5) + 804(354.08 - 8.92)(500 - 33)$$

$$= 51.54 + 129.60$$

$$= 181.14 kNm$$

This is the way to calculate the moment of resistance, that is the moment of resistance of doubly reinforced section is moment of balancing reinforcement section plus the moment of resistance due to the compressed steel. In this first we need to find x_{ulim} and then we need to find out what is the f_{sc}. From this we need to find the stress at particular distance from the neutral axis.

Then using the strain from the table we have find the stress for the corresponding strain. Now by substituting all the parameters we find the value of moment of resistance.

b) An RC beam 300 x 500mm effective in section is reinforced with 804mm^2 and 1963mm^2 of steel at the top and bottom respectively. Find the M_u the beam using M20 and Fe415 as materials d'=33mm.

Solution:

Here also they ask as to find the moment of resistance that is d is 500mm, b is 300mm. The reinforcement with 804mm^2 and 1963mm^2 that is at the top it isA_{sc}= 804mm^2 and at the bottom it will be A_{st} = 1963mm^2. They were asked as to find the moment of resistance using the material M20 and Fe415 with d'=33mm.

i) To find $x_{u\,lim}$:

$$M_u = 0.36 f_{ck} x_{u\lim} b(d - 416x_{u\lim}) + (f_{sc} - f_{cc}) A_{sc}(d - d')$$

$$x_{u\lim} = 0.479d$$

$$= 0.479 \times 500$$

$$= 239.5mm$$

$$d' = 33mm$$

ii) To find f_{sc}:

First we need to find the strain at d' from the top that is nothing but the strain at (x-d') from neutral axis.

$$\frac{0.0035}{x_{u\lim}} = \frac{strain}{x_{u\lim} - d'}$$

$$Stain = \frac{0.0035}{x_{u\lim}}(x_{u\lim} - d')$$

$$= \frac{0.0035}{239.5}(x_{u\lim} - d')$$

$$= 0.00302$$

Now we need to find out what is the moment of resistance M_U. And also we need to find out the stress which is 354.03 N/mm^2. After substituting these values we get the values of M_U as,

$$M_u = 181.14kNm$$

Design of doubly reinforced rectangular sections:

1. **To design the section**

a) An RC beam 200mm x 300mm effective is constructed with M20 and Fe415 as materials d'=30mm. Find the steel required of the factored moment on beam is 74kNm.

Solution:

We know that the moment of resistance of the beam is the moment of resistance of the balanced reinforced section plus the moment of resistance to the compressed section. So first we need to assume our section is a balance singly reinforced section. Then we need to find out the A_{st} for the balanced singly reinforced section.

First assume the given section as balanced singly reinforced section. Here the problem given is b = 200mm and d = 300mm. We are known already that it is a singly reinforced or doubly reinforced section. First we need to find out M_{ulim}.

The given values are b = 200mm; d = 300mm; d'=30mm; and the material M20 and Fe415, M_u=74kNm.Here next we need to check it is a singly reinforced section or doubly reinforced section. To confirm that we need to consider the M_u limit for the given steel,

$$M_{u\lim} = 0.138 f_{ck} bd^2$$

$$= 0.138 \times 20 \times 200 \times 300^2$$

$$= 49.68 kNm$$

So comparing the value of M_u and $M_{u\,lim}$ it is M_u which is the greater one, hence it is a doubly reinforced section. Now to find the A_{st} for balanced singly reinforced section at the neutral axis that is $C_u = T_u$.

$$0.36 f_{ck} x_{u\lim} b = 0.87 f_y A_{st1}$$

$$A_{st1} = \frac{0.36 f_{ck} x_{u\lim} b}{0.87 f_y}$$

$$= \frac{0.36 \times 20 \times 0.474 \times 300 \times 200}{0.87 \times 415}$$

$$= 573.31 mm^2$$

To find excess moment:

$$M_1 = M_u - M_{u\lim}$$

$$= 74 - 49.68$$

$$= 24.32 kNm.$$

To find A_{st2}:

$$M_1 = 0.87 f_y A_{st2} (d - d')$$

$$Ast_2 = \frac{M_1}{0.87 f_y (d - d')}$$

$$= \frac{24.32 \times 10^6}{0.87 \times 415 \times (300 - 30)}$$

$$A_{st2} = 249.975 mm^2$$

To find A_{sc}:

$$M_1 = A_{sc} (f_{sc} - f_{cc})(d - d')$$

$$A_{sc} = \frac{M_1}{(f_{sc} - f_{cc})(d - d')}$$

To find the f_{sc} we need to find the strain at (x_{ulim}– d') from the neutral axis which is equal to 0.002769 and the stress is 353 N/mm^2.

$$= \frac{2432 \times 10^6}{(353 - 8.92)(300 - 30)}$$

$$= 261.78 mm^2 .$$

Total A_{st} is equal to A_{st1} plus A_{st2} that is equal to 826.61mm^2. So this is the way of designing doubly reinforced section.

Summary:

Now we need to summarize this lecture. In this lecture we have learned or we have worked out some of the examples to finding out ultimate moment of resistance for doubly reinforced section. And in the case of design of doubly reinforced section we have worked one problem. From this we can easily design how to design the area of reinforcement for doubly reinforced sections. Doubly reinforcement consists of two A_{st} that is A_{st1} and A_{st2}.

Questions:

1. Find the moment of resistance (Mu) of a beam using following data b=240mm; d=500mm; d'=40mm; Ast = 2140mm^2; Asc=1035mm^2; Concrete M20 and steel Fe415.

2. A beam 300 x 600mm effective in section carries a factored moment of 320 kNm at a section. Find the steel required if M20 and Fe415 are used as materials d'=30mm.

Reference:

- IS 456:2000 plain and reinforced concrete – Code of practice
- Reinforced concrete design by S.N Sinha.