Design of Structures II
Lecture 15

Welcome to the UGC lecture series in B.Architecture. Here, we are having the subject, Design of structures II. In the previous lecture, we have seen how to find out the moment of resistance of the Singly reinforced section and how to find out the area of reinforcement required for the singly reinforced section. In this lecture, we are going to have; Analysis of Doubly reinforced rectangular beam. How to analyze the doubly reinforced sections and how to the moment of resistance of the doubly reinforced section. Using the moment of resistance i.e using the expression, we are going to find out what is the moment of resistance of the doubly reinforced section. Here, first, how to develop the expression for finding the moment of resistance of the doubly reinforced section. Before it, we need to develop a table which shows the stresses and strains for Fe415, Fe500 grade steel at various stress levels, using the stress strain relationship of cold worked deformed bars which is given in IS456 i.e this is the table I have prepared. How do we prepare this table? This is a stress and strain for Fe415 this is a strain and for Fe500 at various stress levels using the common stress strain relationship of cold worked deformed bars. Cold worked deformed bars, here it is Fe415 and Fe500. So based on this table only, we can easily find out what will be the stress at the compression reinforcement i.e Stress at a distance d' from the topmost extreme fibre. So for finding out the stress fsc, in the case of compressive steel we need to first find out what is the strain in the place of d' at the topmost extreme fibre, that is mostly important. So first we need to find out the strain, then using this table, we have prepared based on the stress strain relationship of the cold worked deformed bar, we need to find out the respective stress i.e we need to find out the stress for the respective strain. So we need to find out what is the strain at the level of compressive steel for our problem. Then using the steel by interpolating we need to find out the stress at the respective strain, that is the stress at the level of compressive steel or at the compressive steel. How to prepare this table? That is mostly important.

So here, in the case of doubly reinforced section; we know that doubly reinforced section is the section which has reinforcement both at the top as well as at the bottom, the section is provided a reinforcement at the top and at the bottom, that is called as doubly reinforced section. We all know that under what circumstances doubly reinforced sections are designed. Due to structural or architectural consideration, the structure will have beam insufficient i.e the size of the section will have reduced. Due to the reduction of the size of the section, the section will have insufficient area of concrete in the compression zone to provide for compressive stresses. So, the steel is necessary to provide in the compression zone to resist the compressive stress. When the steel is placed in the compression zone, it becomes doubly reinforced rectangular section.

Here for Fe415 grade steel, Strain at 0.80fy stress level. Strain is;

ϵ_t= fy ÷1.15 E_s

This is the maximum strain.

ϵ_t = fy ÷ 1.15E_s + 0.002

But at the stress level 0.80fy, a corresponding strain i.e

$$= \frac{fy}{1.15Es} + 0$$

It is 0 for the stress level 0.8085. So here, this is the stress strain relationship for Cold formed steel i.e the first one is 0.80fy, it is 0. Another one is, 0.001. Another one is, 0.85fy, this is 0.90fy, 0.003 and another one is 0.007, 0.97fy, 0.95fy and 0.975fy the stress here it is; 0.001 and the maximum strain is 0.002 for F5. The strain here is

= 415 ÷1.15 x 2 x 10^5 + 0

That is the strain here it is,

= 0.00144

and the corresponding stress. The stress for 0.80fy. This is;

= 0.80 x 415 ÷1.15 x 2 x 10^5 + 0.

For the corresponding stress;

= = 415 ÷1.15

This is the stress. The stress here it is;

= 288.7 N/m^2

This is the first level, for 0.85fy stress level. The strain is;

Fy ÷ 1.15 E_s + 0.0001

The corresponding strain here is 0.0001. Initially 4.80fy it is 0. But here we need to substitute,

$= 0.85fy \div 1.15 \times E + 0.0001.$

This is 0.85.

$= 0.85 \times 415 \div 1.15 \times 2 \times 10^{5} + 0.0001$

That is equal to; 0.00163. This is the strain here and the corresponding stress, it is;

$= 0.85fy \div 1.15$

$= 0.85 \times 415 \div 1.15$

$- 306\ N/m^{2}$

This is the way we need to calculate the strain and the corresponding stress and the corresponding strain. For example; Fe500.

For Fe500, first for Stress level 0.80fy. The Strain is normally;

$\epsilon_t = 0.80fy \div 1.15\ Es + 0$

$= 0.80 \times 500 \div 1.15 \times 2 \times 10^{5} + 0$

$= 0.00174$

and the corresponding stress

Stress $= 0.80fy \div 1.15 = 347.8\ N/mm^{2}$.

Then for another stress level. Stress level, I am taking 0.80fy. For 0.90fy, the strain is;

$\epsilon_t = 0.90fy \div 1.15Es + 0.0007$

$= 0.00226$

and the corresponding stress is;

Stress = $0.90fy \div 1.15$

$= 0.90 \times 500 \div 1.15 = 391.3\ N/mm^2$

Next one, for final stress level. For the stress level fy.

$\epsilon_t = fy \div 1.15\ Es + 0.002$

$= 0.00417$

and the corresponding stress here is;

Stress = $fy \div 1.15$

$$= \frac{500}{1.15}$$

$= 434.8\ N/mm^2$

This is the way of calculating the stress and strain for Fe415 and Fe500, separately using the common stress strain relationship of cold formed deformed bar as per IS456:2000. This is the table we have found, now how do we use this table for finding out the moment of resistance as well as, how do we design the section? We will see.

Now here, before that, we need to determine, we have derived and developed the table which shows the stress and strain separately for two grades of steel using the stress-strain relationship of cold form or cold worked deformed bars. Now, we need to determine the ultimate moment of resistance of the doubly reinforced rectangular beam using the limit state concept. Here in this way, we need to find out Xu. Xu normally we know that, how are we finding Xu? At the neutral axis, the total compressive force must be equal to the tensile force. This is the total compressive force and this is the total tensile force and using this, we need find out what will be the resistance of the doubly reinforced rectangular section. Now, how do we find out?

So here this is a beam. Here it is reinforced at the bottom and this is Ast and the depth of the section is d and the breadth of section is b and it is also reinforced at the top i.e at the compression zone which is placed at d'. This is Asc, so it is a doubly reinforced rectangular section and this is the main. I take this as the neutral axis, this is a strain diagram. Maximum strain at the topmost extreme fibre is Є Cu i.e 0.0035. Maximum strain at the tension zone is;

$$\epsilon_{tu} = fy \div 1.15Es + 0.0002$$

So here, this is Xu and this will be (d - Xu) and the stress diagram here it is, this is a stress diagram. So up to the strain, 0.0002, the stress is uniform and thereafter the stress will be varying and Tu be the total tensile force which is offered by the Tensile reinforcement and here the maximum stress at the topmost extreme fibre is 0.446fck and the depth of this parabolic section is 0.57Xu and the depth of the rectangular portion is 0.43 Xu, we have already divided the singly reinforced section and I am taking Cu_1 as the total compressive force offered by the whole concrete area above the neutral axis which is acting at 0.416Xu from the topmost extreme fibre and the corresponding tensile forces Tu1 and the distance between these two force is called Lever arm Z1 i.e (d - 0.416Xu) and since the steel is placed at the compression zone which is at a distance d' from the topmost extreme fibre. There is also another force Cu_2which is offered by the compressive steel and the corresponding tensile forces is Tu_2 and the distance between these two forces and another lever arm is Z_2 i.e (d - d') Now we need to find out Xu.

At the neutral axis, we all know that Cu must be equal to Tu. Here, $Cu_1 + Cu_2 = Tu$. Commonly it is Tu i.e Ast. What is Cu_1? Cu_1 is the total compressive offered by the whole concrete area above the neutral axis, that is we have direct;

$$0.36fck\ Xub +$$

What are the compressive forces offered by the area of reinforcement Asc? So first, force is normally stressed into area. Stress area x Stress is fsc. fsc is stress at the compressive steel at a distance of d' from the topmost extreme fibre. So here, we have placed the steel, after removing the concrete, we have provided the steel. So we need to deduct the stress in the concrete i.e fcc at the level of compressive steel. Stress in concrete at the level of compressive steel to be deducted from the stress in steel. Since some amount of concrete is removed from the beam, in place of that we have provided steel. So, we need to first deduct the compressive stress in concrete and we have provided the compressive steel that the stress in concrete has to be deducted from the stress in compressive steel. So, this is the total stress and if it is multiplied by the area of steel, we get the Cu_2. That has to be added;

$0.36fck\ Xub + Asc\ (fsc - fcc) = 0.87fy\ Ast.$

So here, in the case of the doubly reinforced beam. The doubly reinforced beam is an over reinforced beam. If Xu is greater than Xu_{lim}, it is doubly reinforced beam or Mu is greater than Mu_{lim}. How to design this section? Here, I have already told you that if Mu is greater than Mu_{lim}, for the excess moment,

M1 i.e $Mu_1 = Mu - Mu_{lim}$

For the difference in this moment, we need to provide the reinforcement at the compression zone and the section will become doubly reinforced section.

How do we design a section? First, we ought to assume that the section is a balanced singly reinforced section i.e Mu_{lim}. Up to the Mu_{lim}, we need to design the section that has to be provided at the tensile zone, that is called as Ast1. Ast1 is the area of reinforcement required for the balanced singly reinforced section when $Mu = Mu_{lim}$. Then we have found Mu1, $Mu > Mu_{lim}$, Mu_1 is $Mu - Mu_{lim}$. For that we are providing Asc and for balancing this, we'd like provide again As to the additional tensile reinforcement to the tension zone. So, Ast1 is the area of reinforcement required for the balanced reinforced section. Ast_2 is the additional tensile reinforcement to balance Asc which has been provided at the top. What is the moment of resistance at the beam?

Moment of resistance of the beam is? What is the total moment of resistance of the doubly reinforced section? Moment of resistance of a balanced singly reinforced section + the moment of resistance due to the compression reinforcement. What is the moment of resistance $Cu_1 \times Z_1 + Cu_2 \times Z_2$. But how to find, the moment of resistance is;

$$= 0.36 fck\, Xu_{lim}\, b \times (d - 0.416 Xu_{lim})$$

This is the moment of resistance of the balanced singly reinforced section + moment of resistance due to compressive reinforcement i.e Cu_2.

$$= 0.36 fck\, Xu_{lim}\, b \times (d - 0.416 Xu_{lim}) + Asc\,(fsc - fcc)\,(d - d')$$

Cu_2 is the Asc (fsc - fcc) i.e the stress x area x the lever arm i.e the force into the distance covered. Moment of resistance of doubly reinforced section is moment of resistance of a balanced singly reinforced section + the moment of resistance due to the compressive reinforcement. In another expression, it can also be written as;

$Mu = Tu_1 \times Z_1 + Tu_2 \times Z_2$

$= 0.87fy\ Ast\ (d - 0.416Xu_{lim})$

This is the moment of resistance of balanced singly reinforced section, plus the moment of resistance due to additional tensile reinforcement.

$= 0.87fy\ Ast\ (d - 0.416Xu_{lim}) + 0.87fy\ Ast_2\ (d - d')$

So, using these two expressions, I am writing on this slide;

$Mu = Cu_1 \times Z_1 + Cu_2 \times Z_2$

$= 0.36fck\ Xu_{lim}\ b\ (d - 0.416Xu_{lim})$

This is the moment of resistance of a balanced singly reinforced section.

$= 0.36fck\ Xu_{lim}\ b\ (d - 0.416Xu_{lim}) + Asc\ (fsc - fcc)\ (d - d')$

Now, we can find out fsc. How do we find out fsc? fsc we need to find out the stress at the level of d' at the topmost extreme fibre. So, how to find out the stress in this reinforcement? From the diagram, we can easily find out what is the strain at the level of compressive steel from the strain diagram i.e at the depth of Xu from the neutral axis, the strain is 0.0035. I am going to find out at a depth of Xu - d', what is the strain of d' at the topmost extreme fibre? From the strain, I can use this tabular column, I can find out the stress. So, fsc we can easily find out, fsc can be found out from the tabular column. Once you find out the strain of d' at the topmost extreme fibre, we can find out what is the stress in the compression reinforcement i.e at a distance d' from the topmost extreme fibre, using the table we have derived in the lecture.

Next, what is fcc? How do we find out fcc? fcc is the stress in concrete at the level of compressive steel i.e at a distance d' from the topmost extreme fibre. So here, this is d', this is the place where we have to find out fcc. Now this is up to 0.43Xu, the stress is uniform in the concrete. Here, the 0.43Xu is greater than d' or d' is very much lesser than 0.43Xu upto 0.43 Xu, the stress is 0.446fck. What about the stress at the level of d' from the topmost extreme fibre? It is also 0.446fck i.e

fcc = 0.446fck

The stress is uniform throughout the rectangular portion of the stress block i.e 0.446fck i.e upto 0.43Xu from the topmost extreme fibre. But d' is very much lesser than 0.43Xu. So, fcc at d' from the top here it is, 0.446fck. So, this is the way of calculating the moment of resistance for the doubly reinforced section. So here, we know fck, we can easily find out Xu_{lim}, we know b, we know d, we know Xu_{lim}, we know Asc, we are going to find out the strain d' at the topmost extreme fibre from the strain diagram and we can select the stress from the table for the respective strain i.e fsc and we can find out what is fcc. fcc is the stress in concrete at the level of compressive steel i.e 0.446 fck, we know d, we know d'. Using this expression, we can easily find out what will be the moment of resistance of the section. So, next one is, we need to find out.

Let us summarize this lecture. In this lecture we have seen, the analysis of doubly reinforced rectangular beam. Here, we have derived the moment of resistance of the doubly reinforced section and also we have developed the table which shows the stress and strain for Fe415, Fe500 separately, using the stress strain relationship of word form, the common stress-strain relationship of cold worked deformed bars from IS456:2000 and the expression for the moment of resistance for the doubly reinforced section; one is Cu_1 x Z_1 and Cu_2 x Z_2 and another formula is Tu_1x Z_1 and Tu_2 x Z_2. So here, the first formula is 0.36fck Xu_{lim} b (d -0.416Xu_{lim}) + Asc x fsc - fcc x d - d'. Next one is, another formula is Tu x Z; 0.87fy Ast1 (d - 0.416Xu_{lim}) + 0.87fy Ast_2 x d - d'. From these two expressions we can find out the moment of resistance of the doubly reinforced section and fsc we can easily find out, first we need to find out the strain of the problem, then fsc can be easily found from the respective strain from the table. Questions, the most important question here is, we need to know how to develop a table showing the stresses and strain separately for Fe415 and Fe500 steel at various stress levels, using stress-strains, common stress strain relationship of cold worked deformed bars as per IS456:2000, this is very important. Without knowing developing and this table, we cannot design or find out the moment of resistance of the doubly reinforced section. Next, we must know how to derive the moment of resistance of the doubly reinforced section and with regard to the references, we can refer; IS 456:2000 i.e Plain and reinforced concrete - Code of practice and the book which is written by S.N Sinha it is, Reinforced concrete design. With this we come to the end of this lecture, Thank you!