Design of Structures II
Lecture 14

Welcome to the UGC lecture series in B.Architecture. Here, we are having a Design of structures II. In the previous lectures we have seen; how to find out the moment of resistance of a singly reinforced rectangular beam, then we have found; the depth of neutral axis and the limiting value of depth of neutral axis and also, we have found the moment of resistance of the respective section and also we have found the ultimate moment of resistance of the rectangular beam and using this we have also analyzed the singly reinforced rectangular beam and we have found the moment of resistance of the section. Now in this lecture, we are going to design singly reinforced rectangular beam i.e to design the size of section as well as the reinforcement required for it or by giving the size of the section, how to find out the area of reinforcement required for it by using the limit state design method. So here, the first problem here is, A rectangular section which is given in the problem i.e the size of the beam here is - 360 by, since it is a singly reinforced rectangular beam the reinforcement is provided in the tension zone, the effective depth of beam is - d =500, b = 360mm and they have given the factored moment. Normally a factored moment means that the design moment which has already been multiplied by the factor of safety. Design moment normally Md which has again been multiplied by 1.5 will give you the ultimate moment of beam i.e the moment Mu, which is equal to 175 kNm, this is divided by the problem and now how to find out how to find out this area of reinforcement required for it. Here, I have already told you that we already found out the moment of resistance of the beam i.e

$Mu = Tx Z$ i.e

$= 0.87fy\ Ast\ (d - 0.416Xu)$

$= 0.36fck\ Xu_b\ (d - 0.416Xu)$

So using this expression we can find b and d. Using this expression we are going to find out what is Ast. So, the size of the section has already been given and first we need to come to know that whether our section a singly reinforced or doubly reinforced or under reinforced or a doubly reinforced section. There is no over reinforced section, if an over reinforced section comes, we need to design it as a doubly reinforced section by placing the steel in the compression zone to resist the required compressive stress which is developed in the concrete, when you reduce the size of the section.

Now here, the factor moment is given in the problem. Now, first we know that the section is coming under under reinforced section or doubly reinforced section. So for this, to find Mu_{lim}; we can also divide our section as under reinforced or doubly reinforced by comparing Mu with

Mu_{lim}. So if Mu is less than Mu_{lim}, it is under reinforced section, that we have studied. If Mu > Mu_{lim}, it is a doubly reinforced section. That we have studied. So first to find out, Mu_{lim} for the given grade of steel;

Grade of steel is given in the problem i.e Grade of steel is Fe500, for 500 we have found the Mu_{lim} is $0.133fck\ bd^2$. So, 0.133fck given in the problem i.e M20 grade concrete i.e M20. B is 360. Mu_{lim} for here it is; 268.203 kNm. Have you compared Mu with Mu_{lim}?

$Mu_{lim} = 0.133fck\ bd^2$

$= 0.133 \times 20 \times 360 \times 500^2$

= 199.5 kNm

So here, $Mu < Mu_{lim}$. Hence it is, under reinforced section.

Now, moment of resistance for under reinforced section is, to find out area of steel.

Mu = Tu x Z

= 0.87fy Ast (d - 0.41bXu)

Here we know Mu, Mu is given in the problem 175kNm. fy i.e 500 nm^2. Ast we need to find out. d is given in the problem, what is Xu?

Xu is; To find -

Xu is normally $0.87fy\ Ast \div 0.36fck\ b$

Here, fy is known, fck known, b is known, we have to know the Ast. Xu has to be found in terms of Ast i.e fy is

$Xu = 0.87fy\ Ast \div 0.36fck\ b$

$= 0.87 \times 500 \times Ast \div 0.36 \times 20 \times 360$

i.e in terms of Ast

= 0.20 Ast

Now, we need to substitute Xu in Mu

Mu = 0.87fy Ast (d - 0.416Xu)

175kNm = 175 x 10^6 Nmm

This equation; Mu = 0.87fy Ast (d - 0.416Xu), is expressed in Nmm. That's why I am converting this;

175 x 10^6 = 0.87 x 500 x Ast (500 - 0.416 x 0.20 Ast)

So here, we are going to get a quadratic equation;

Ast^2 - 5973.6Ast + 4806372 = 0

This is the quadratic equation that we need to obtain. By starting this quadratic equation, normally quadratic equation by solvind quadratic equation, I am going to get 2 routes. Since it is under reinforced section, we need to select the minimum route. So here, the minimum route of this quadratic equation is -

Ast = 958.32mm^2.

This is the way of calculating Ast required for the beam. From this, we can assume the diameter of the reinforcement if the diameter of the bar is 16mm, number of bars which is

Number of bars = 958.32 ÷Area of 1 reinforcement π ÷4 $(16)^2$

We need to find out the number of bars. This is the way of designing the singly reinforced rectangular beam. The moment is given in the problem. For example the beam is, a singly supported beam which is given in the problem, if it is subjected to some kind of loading. So, for example it is; 15kN/m. We can easily find out the moment of the beam if the span of the beam is normal. Mu = 1.5. This is the partial safety factor for the load x 15 x 6^2 -

Mu = 1.5 x 15 x 6^2 ÷8

Using this moment, we are going to find out the Mu_{lim} for the respective of grade of concrete used in the design. We have to find out Mu_{lim}, when you compare Mu_{lim} with Mu, we conclude whether our session is balanced section or unbalanced section, under reinforced or doubly reinforced. If Mu < Mu_{lim}, it is under reinforced section and for under reinforced section, the equation is 0.87Ast x d - 0.416Xu. From this we are going to find out what is Xu. Xu in terms of Ast, then you substitute in this expression. Then you have a quadratic equation by solving this quadratic equation, we are getting two routes, we need to select this minimum routes since it is under reinforced or singly reinforced section. This is the way we need to design.

Next, we need to see the second problem i.e again a rectangular beam is given here. Here the size of the singly reinforced beam is, 230 (b). Here reinforcement which is provided at the bottom, here d is = 650mm and the moment of resistance Mu is 195kNm. The factored moment is given in the problem. If it is factored moment is taken as Mu i.e 195, find the area of steel required. If concrete M20 and Fe415 are used. M20 and Fe415 as materials. fck for M20 is $20N/m^2$ and fy for given grade of steel is 415 N/m^2 . Find the area of steel is required. First our section, we need to conclude, we first need to find out what is Mu_{lim} for given grade of steel.

To find Mu_{lim}: first we need to finalize whether our section is under reinforced or over reinforced. The moment is given in the problem, Mu. Now, we need to find out the Mu_{lim} for the given grade of steel. So Mu_{lim} for Fe415 is 0.138 fck bd S^2.

Mu_{lim} = 0.138 fck bd^2

= 0.138 x 20 x 230 x 650^2

= 268.203kNm.

This is the way we need to find out the Mu_{lim}. Now, you compare Mu_{lim} with given Mu. So here, Mu is less than Mu_{lim}. So, it is Singly reinforced rectangular section. Singly reinforced rectangular section (or) under reinforced section. So, to find out Ast.

For Ast of under reinforced, balanced section, Tu x Z i.e 0.87fy Ast (d - 0.416Xu) Now, we need to find out Xu.

$Xu = 0.87fy\ Ast \div 0.36fck\ b$

$= 0.87 \times 415 \times Ast \div 0.36 \times 20 \times 250$

$= 0.218\ Ast$

I have found Xu in terms of Ast. I am going to substitute in this expression.

$Mu = 0.87fy\ Ast\ (d - 0.416Xu)$

$195 \times 10^6 = 0.87 \times 415 \times Ast\ (650 - 0.416 \times 0.218Ast)$

So here, after solving this expression, I am getting a quadratic equation of;

$Ast^2 - 7168.07\ Ast + 5.96 \times 10^6 = 0$

This is the quadratic equation and now the quadratic equation we can solve. For example;

$ax^2 + bx + c = 0$

The coefficient of x is a, the coefficient of x is b and the constant c, we need to find out x.

$x = -b +/- \sqrt{b^2} - 4ac \div 2a$

This is the root of quadratic equation. This is the formula to find the roots of the quadratic equation using this expression. So, here I am finding Ast. When you solve this equation, you get two roots, so we are going to get 2 roots. The minimum root here it is 960.05 mm^2 since it is under reinforced or singly reinforced section. So, this is the way we need to calculate the area of reinforcement required for it. The continuation of this problem, here also we are going to choose another two problems to find out the moment of resistance as well as the design of the section. So here, I am going to do the, how to find out the reinforcement required for the beam by using another problem i.e a rectangular beam. Here, the size of the beam is given in the problem. We are not sure whether it is under reinforced or doubly reinforced section, we have to. The depth of the section is 500mm and the Factored moment they have given. The factored moment Mu =155kNm and M20 and Fe500 is given in the problem i.e materials. So here, we are going to design the section. First we need to find out what is Mu_{lim}, I am explaining the

steps as well. I am explaining the steps of area of reinforcement here. It is another model problem. Mu_{lim} for the given grade of steel is;

$Mu_{lim} = 0.133fck\ bd^2$

In mild steel, it is; $0.149fck\ bd^2$ and $0.138fck\ bd^2$ fe415. For Fe500 it is $0.133fck\ bd^2$. So, Mu_{lim} we need to compare with the given moment if Mu is less than Mu_{lim}, we need to finalize it is under reinforced or singly reinforced section. So, if it is under reinforced section,

$Mu = Tu \times Z$

$= 0.87fy\ Ast\ (d - 0.416Xu)$

So, we need to find out Xu in terms of Ast.

$Xu = 0.87fy\ Ast \div 0.36fck \times b$

So, we need to find out Xu in terms of Ast and substitute here and we need to find out what is the reinforcement required for it i.e by substituting;

$Mu = 0.87fy\ Ast\ (d - 0.416Xu)$

From this expression, by substituting Xu in terms of Ast, we need to find out a quadratic equation that we have not formed plus the constant which is equal to zero. By solving this quadratic equation we can select a minimum root, that will be the Ast required for it. In the first problem i.e Mu, they have asked us to find out Mu, we ar going to do this i.e

b = 230mm

It is a singly reinforced beam.

d = 600mm

and Area of reinforcement is given in the problem i.e 942 mm^2 and it also asks us to find out what is the safe load for finding out the moment of resistance. So, M20 and Fe415 is given in the problem and they have asked us to find out what is the moment of resistance and safe load i.e UDL of the beam. So, first to find out Xu. That we all know, the steps to find out the moment of resistance i.e

Xu = 0.87fy Ast÷ 0.36fck b

So, we need to find out what is Xu. Then to find out Xu_{lim}. Xu_{lim} for the given grade of steel is;

0.479d

So we need to find out. So, here if Xu is less than Xu_{lim}, it is an under reinforced section. So,

Mu = Tu x Z

Tu we need to calculate i.e

= 0.87fu Ast (d - 0.416Xu)

So, here it is by substituting fy = 415, Ast = 942, d - 600, Xu we have found, we need to find out what is the moment of resistance of the beam. So, in this section we have seen many number of questions, many number of problems, we have designed the moment of resistance of the section, as well as, we have found how to out the moment of resistance of the under reinforced section as well as how to design the section we know. In the case of singly reinforced section, normally most of the sections have to be designed as under reinforced because under reinforced section I have already explained it to you. Under reinforced section is the most safest section since the nature of failure here is a ductile failure or it is a tensile failure. This failure normally indicates the failure of the section, so we can easily rehabilitate this section. So, in order to design whether it is a singly reinforced section, that section should be designed as an under reinforced section.

Now, I shall summarize the lecture and what we have seen so far. The case we have seen, the design of a rectangular beam as well as the moment of resistance of a singly reinforced rectangular beam and how to find out the moment of resistance of the beam in the case of singly reinforced i.e first find out Xu and then you find out Xu_{lim} for various grades of steel. Then you compare Xu with Xu_{lim}, we can easily find out whether our section is under reinforced or doubly reinforced section or it has to be redesigned or not. Then we need to use the formula for the respective sections, we can find out the moment of resistance of the section. Xu limit for Mild steel is 0.531d, Xu_{lim} for Fe415 is 0.479d, Xu_{lim} for Fe500 is 0.456d. So, by comparing Xu_{lim} with Xu, if Xu is lesser than Xu_{lim}, we conclude that our section is under

reinforced or singly reinforced. If it is, it has to be redesigned, it acts as a doubly reinforced section. Here also, we have designed the section. The design of the section is the most important thing. So, we have used two formulas, we have found two formulas. One being 0.36fck Xub (d - 0.416Xu), that is the formula used to design the section i.e b and d. Anyone, b we need to assume that we know the standard dimension is 230mm, 300mm, 340mm, 380mm, these are all the standard dimensions of the beam. If it is required, we can also change it. It is needed we can change this b as 250mm, for the respective beam, we can easily find out what is d using that formula. By assuming the b. Then, Ast we can find out from another formula i.e Tu x Z i.e 0.87fy Ast (d - 0.416Xu). So here, the Xu in terms of Ast, we can easily find out what is Ast required for singly reinforced rectangular section.

The questions we have already asked, we have already worked out. Anyhow, we can solve these problems i.e to find out the moment of resistance of the beam. Find the Mu of the rectangular beam of size 230mm x 600mm effective which is reinforced with 942mm^2 of steel at tension zone. Use M20 concrete and Fe415 steel, we can also change the grade of concrete. We find out the moment of resistance, we find out whether is under reinforced or doubly reinforced. Then you can easily find out what will be the safe load on the beam by deducting the load from the moment of resistance with the by deducting the sulphate of the beam, we can easily find out safe load.

In order to design, there is another problem given; A beam 300mm x 500mm effective size carries a factored moment of 155kNm. If concrete M20 and steel Fe500 are the materials, we can easily find out what will be the steel area required for this section. We can also change the dimension, we can calculate the subpractice on working on this problem, by doing a number of problems, we can easily familiarize how to design the singly reinforced rectangular beam. In this lecture, I have referred; IS 456:2000 Plain and reinforced concrete - code of practice and also another book by S.N Sinha i.e Reinforced Concrete design and I have already told you that both this S.N Sinha has explained theoretically about the design of reinforced concrete members as well as analytical expressions have also been derived in detail. By referring this book, we can solve several problems by gaining practice on singly reinforced rectangular section. Let us come to the end of this lecture, Thank you!