Design of Concrete Structures II
Lecture 13

Welcome to the UGC lecture series in B.Architecture. Here, we are having a look on the subject, Design of structure II. In the previous lectures we have seen how to calculate the depth of neutral axis for the singly reinforced rectangular section or in general and also, we have found the limiting value of depth of neutral axis. Now, in this lecture we are going to find out the Ultimate moment of resistance of the sections.

So, in the previous lecture we have found the Xu i.e the Actual depth of neutral axis and also the limiting value of depth of neutral axis by comparing one with another, we can find out whether our section is under reinforced or balanced section or over reinforced section. But in the case of Limit state method the over reinforced section is completely avoided. When it comes, it has to be redesigned as a doubly reinforced section since in an over reinforced section, we are using more amount of steel. Automatically the area of concrete is very less, in order to supplement the concrete in resisting the compressive stress, you need to provide the reinforcement on the top. So, this section will become a doubly reinforced section when it is placed when the reinforcement takes place both at the top as well as the bottom.

Now, here we are going to analyze ; the Singly reinforced section to find out the moment of resistance of the section and also we are going to find out what will be the limiting moment of resistance of the section i.e Mu is limiting. Mu_{lim} or Mu_{max}

So here, Moment of resistance which is normally moment, moment is normally the force offered by the compressive force and the force offered by the tensile force.

Force x the Respective Lever arm.
That is the Moment of resistance we are going to find out i.e the Ultimate moment of resistance of the beam. So here, for this I am taking this as the beam i.e a singly reinforced rectangular beam of size 'b' and the depth of the section here it is 'd' and here, the strain diagram. You think this as the neutral axis, this is a strain diagram. Maximum strain is;

EPSILON Cu= 0.0035 and

Epsilon Tu = fy ÷1.15Es + 0.002

The stress diagram here it is, this is a stress diagram and this is the tensile force offered by the wohle concrete area and this is a; Rectangular stress block, the depth of Rectangular stress block is 0.57Xu and this one is, 0.43Xu and the depth of neutral axis here it is, Xu. That we all know. Now, we need to find out what is the moment of resistance of the beam? We have found the total compressive force offered by the whole concrete area i.e;

Cu = 0.36fck Xub

and the total tensile force offered by the tensile reinforcement is;

Tu = 0.87fy Ast

and this is the total compressive force which is offered by the whole concrete area which is acting at the CG of the stress black from the topmost extreme fibre. As in the case of working stress method, the distance between the resultant compressive force and the resultant tensile force is the Lever arm, that is equal to Z or you can take this as $\bar{y}$. I am taking this as Z i.e d -$\bar{y}$. Now how to calculate? First to find out $\bar{y}$ i.e distance of C.G of Compressive Stress Block from Top. $\bar{y}$ is the distance of C.G Stress block from the topmost extreme fibre i.e Total compressive force offered by the whole concrete area above the neutral axis. $\bar{y}$ is the distance of the C.G of Compressive stress block from the top.

$\bar{y}$ = A1y1 + A2y2 + ... ÷ A1 + A2 + ...

Since we are having two areas, we are going to have;

A1y1 + A2y2 ÷ A1 + A2

A1 is the area of the compressive stress block. Maximum compressive stress is 0.446fck on the top. Now, we need to first find out what is the area, A1 is the area of the compressive stress block i.e Area of the rectangular portion of the stress block.

= 0.46fck x 0.43Xu

The distance of C.G of the stress block from the top is;

= 0.46fck x 0.43Xu x 0.43Xu/2

Since we are finding the C.G of the overall stress block from the top, we are finding the C.G of the individual component from the top, this is A1y1, next A2 is;

= [0.46fck x 0.43Xu x 0.43Xu/2] + 2/3 x 0.57Xu x 0.446fck x

What is the C.G of this parabolic one? It is 3/8th of 0.57Xu. So, since we are finding the vibe of the topmost extreme fibre we need to calculate what is the C.G of this parabolic section from the top i.e

= [0.46fck x 0.43Xu x 0.43Xu/2] + 2/3 x 0.57Xu x 0.446fck x (0.43Xu + 3/8th x 0.57Xu)

This is A2 and y2 divided by;

= [0.46fck x 0.43Xu x 0.43Xu/2] + 2/3 x 0.57Xu x 0.446fck x (0.43Xu + 3/8th x 0.57Xu) ÷ (0.446fck x 0.43Xu) + 2/3 x 0.57Xu x 0.446fck

= 0.416Xu = 0.42Xu

as given in IS 456 i.e 0.42Xu

This is a way of calculating the disturbance of C.G of the compressive force which is offered by the whole concrete area above the neutral axis i.e the C.G of the compressive stress block from the topmost extreme fibre. Once you find out C.G, we can easily find out what is the moment of resistance.

The Ultimate moment of Resistance i.e Mu. Normally Mu is if x is less than Xc, it is under reinforced. If X > Xc or if X = Xc, it is balanced section. There is no over reinforced section in the case of limit state method. So, one is Tu x Z

Mu = Tu x Z
= 0.87 fy Ast (d - $\bar{y}$)
= 0.87fy Ast (d - 0.416Xu)

This is the Mu of the singly reinforced section which is offered by the tensile reinforcement. Next, Mu above the neutral axis which is offered by the compressive force is Cu x Z i.e

Mu = Cu x Z
= 0.36fck Xub (d -$\bar{y}$)
= 0.36fck Xub (d - 0.415Xu)

This is another expression which is used to find out the moment of resistance. There have in two different equations; in case of design of RCC elements or RCC beams, we need to design the size of section b and d and also we need to design area of reinforcement. So in order to design these three parameters, we can assume b and we can find out d by assuming d and also we can find out Ast. So, we need to have two different equations i.e two equations to find out these two parameters d and Ast. Using this expression we can easily find out what is Ast. Using this expression we can find out what is d by assuming b. Once you know the external moment, so by equating the external moment to the internal moment you can easily design the size of the section as well as the area of reinforcement required for the moment of resistance.

What is the maximum moment of resistance? This is Mu. We have already found out the Xu, Xu is the actual depth of neutral axis and also we have found the limiting or the maximum depth of neutral axis which is used for finding out whether our section is under reinforced or our section is a balanced section. Now here, in this case also we have found the moment of resistance, we are also going to find out the Maximum moment of resistance i.e Mu_{lim} or Mu_{max} for the various grades of steel.

To find out Mu_{lim} or Mu_{max} (limiting values of Mu) This is also according to the grade of steel, first for Mild steel. Here in this case, I am going to use this as an expression to find out the Mu_{lim} for our steel because so here when you find out the moment of resistance in this expression, it is used to find out the Maximum Mu_{lim}, we can equate these two equations to find out their effective depth of the section. So here, this is Xu. X < Xu, when it reaches X = Xu, we can use both the expressions to find out the moment of resistance for the balanced section. So here, when it reaches this one, I am using this as an expression to find out that Mu_{lim}.

Mu_{lim} = 0.36fck Xu_{lim}

So here, Xu is Mu. Xu is normally for finding out Mu. Once you substitute Xu_{lim}, this equation becomes;

Mu_{lim} = 0.36fck Xu_{lim} x b(d.0.416 Xu_{lim})

= 0.36fck x 0.531d x b(d - 0.416 x 0.531d)

After simplifying this, after multiplying all these things we can get;

= 0.149fck bd^2

Then for Fe415 i.e 0.149fck bd^2 . Then for Fe 415;

Mu_{lim} = 0.36 fck Xu_{lim} b(d - 0.416Xu_{lim})

$= 0.36fck \times 0.479d \times b(d - 0.416 \times 0.479d)$

which is after simplifying this point;

$= 0.138fck\ bd^2$

In a similar manner, Mu_{lim} for Fe500. $Mu_{lim} = 0.36fck\ Xu_{lim}\ b(d-0.416\ Xu_{lim})$

$= 0.36fck \times 0.456d \times b(d - 0.416 \times 0.456d)$

This we have found, here it is;

$= 0.133fck\ bd^2$

We can also come to conclude, here in this case by comparing Xu with Xu_{max}, we can conclude our session is a balanced or under reinforced session or a doubly reinforced session. Here also, we can compare.

$Mu < Mu_{lim}$

$Xu < Xu_{lim}$

It is under reinforced section. We can conclude. We know the Mu. Mu is an external moment, this one is the internal moment. External moment for simply supported beam which is subjected to UD has to be multiplied by 1.5, we can get Mu.

Mu_{lim} for the different grades of steel, it depends upon the type of the steel used in the design we can easily find out what is Mu_{lim}. Then we compare Mu i.e external moment, we can conclude whether our section is under reinforced or over reinforced. If Mu is less than Mu_{lim}, we can conclude that it is an under reinforced section and also Xu is greater Xu_{lim}, it is an over reinforced section but it is not considered in the case of limit state method, that has to be redesigned as doubly reinforced section. Mu is greater. How to define the doubly reinforced section. Listen in the case of doubly reinforced section, what is the moment of resistance of a doubly reinforced section? Moment of resistance of the doubly reinforced section is the moment of resistance of the balanced reinforced section plus the moment of resistance due to the compressive reinforcement which is placed at the top. At what moment do we need to design the area of compressive reinforcement; $Mu - Mu_{lim}$, for the excess moment we need to provide the compression reinforcement and to balance this one, we need to provide additional tensile reinforcement at the bottom i.e Ast1 + Ast2 and this one is Ast. That section will become doubly reinforced section. This is the thing we have seen so far to find out the limiting value of the moment of resistance. Now, we need to do some problems on calculating the moment of resistance of singly reinforced rectangular section. So first problem, find the moment of resistance that is ultimate moment of resistance of the rectangular beam of size 300mm x 650mm effective which is reinforced with $942mm^2$ of steel at tension side. Use M20 concrete and Fe415 steel. Find also the safe load on the beam if the effective span is 6m. Now here, the rectangular beam is given in the problem i.e this is the area of tensile steel. So, it is a singly reinforced section; d = 500mm and b = 300mm and the area of steel is 940^2 i.e Ast = $942mm^2$. So effective which is depth of the section is use; M20 Grade Concrete and Fe415 grade steel. Given that b = 300mm, d=500mm, Ast = $942mm^2$ and fck for M20 Grade Concrete is = 20 N/mm^2 , fy for Fe415 grade steel is = $415N/mm^2$. They have asked us to find out what is the moment of resistance of the section and also the safe load. So here, the first step is to find Xu. Xu can be found out at the neutral axis.

Cu must be = Tu i.e

0.36fck Xub = 0.87fy Ast

First we need to find out whether our section is under reinforced or over reinforced i.e doubly reinforced.

Xu = 0.87fy Ast ÷0.36fck b

= 0.87 x 415 x 942 ÷ 0.36 x 20 x 300

= 157.46mm

This is the depth of neutral axis and next step;

To find out Xu_{lim} for given grade of steel is;

Xu_{lim} = 0.479d

= 0.479 x 500

= 311.35mm

So here, Xu is less than Xu_{lim}. Hence, it is under reinforced i.e under reinforced section X is less than Xu_{lim} i.e under reinforced section. The moment of resistance for under reinforced section is;

Mu = Tu x Z
= 0.87fy Ast (d.o.416Xu)

After substituting all the values, we will get;

Mu = 198.72knm

Then the second problem. We move on to the second problem i.e Find the moment of resistance of the rectangular section i.e of 225mm x 700mm effective which is reinforced with $1100mm^2$ of steel at tension side. Use M20 concrete and Fe500 steel.

So here, we need to find out the safe load; Safe load after deducting the sulphate of the beam. First we have to find out the sulphate of the beam. Sulphate of the beam is 0.3. Here d = 500, I'm assuming 20mm diameter bar, it is;

D = 500 + 30 + 20/2

= 540mm

Sulphate of the beam = 0.3 x 0.54 x 25
= W1 Kn/m

This is the sulphate of the beam, it is some Kn/m. Now, we have calculated the bending moment.

$Mu = Wl^2 \div 8$

$198.72 = W \times 6^2 \div 8$

This W again has to be divided by 1.5, we can get the load that has to again deducted by the sulphate of the beam, we get the Safe load.

$Mu = Wl^2 \div 8$

i.e when you equate this one, I am dividing again by 1.5 to get the load and that has to be again subtracted by the sulphate of the beam, we can get the safe load acting unit. Then next problem; Mu i.e Size of the beam is given in the problem, it is given that;

b = 225mm
d = 700mm
Ast = 1100mm^2
Grade of concrete is M20 and Fe500

So fck here is 20N/mm^2
fy = 500N/mm^2

Find the moment of resistance of the beam. So here, first to find Xu.

Xu = 0.87fy Ast÷ 0.36fck b

= 0.87 x 1100 x 500 ÷0.36 x 20 x 225

= 295.37mm

Xu_{lim} for given grade of steel for Fe500, we have already found is 0.456d

Xu_{lim} = 0.456d

= 0.456 x 700

= 319.2mm

So here Xu is again less than Xu_{lim}. It is under reinforced. Then Mu for under reinforced section is;

Mu = Tu x Z
= 0.87fy Ast (d-0.416Xu)
= 0.87 x 500 x 1100 (700 - 0.416 x 295.37)
= 276.15 knm

This is the way of calculating the moment of resistance of the singly reinforced rectangular beam. We have seen two problems from this we can easily understand how to calculate the moment of resistance of the respective section. First we find out what is the depth of neutral axis, next we find out what is the limiting value of the depth of neutral axis. We need to compare one with another, then we find out whether our section is balanced/ under reinforced/ over reinforced section or it has to be designed as doubly reinforced section when it is over reinforced section.

Now we come to the Summary of this lecture; In this lecture we have seen; analysis of Singly reinforced rectangular beam. In this moment we have found; Ultimate moment of resistance of the singly reinforced rectangular beam. Then also we have found the limiting values of Moment of resistance. So in this case, we have seen the limiting moment of resistance of the various grades of steel which are used in the design i.e for Mild steel it is 0.149fck bd^2 and for Fe500 it is 0.133fck bd^2 . So we can also compare Mu with Mu_{lim}, we can conclude whether our section is balanced or unbalanced section. Here also, as by comparing Xu with Xu_{lim}. Once it is Mu is less than Mu_{lim}, it is basically an under reinforced section. If Mu is greater than Mu_{lim}, it is an over reinforced section, it has to be designed as doubly reinforced section by providing the reinforcement of the compression zone to supplement the concrete, the X is compressive Stress.

Questions; So in this lecture, determine the expressions for the Ultimate moment of resistance (Mu) of the singly reinforced rectangular beam. We need to know how to calculate the ultimate moment of resistance of the section i.e analytically we must know it, that's why I have asked this question. Then, Find the limiting values of Mu for the various grades of steel. In order to handle these values, we must know how to calculate manually the ultimate or limiting values of the moment of resistance of the beam. Then the case of references for this lecture, you can refer; IS 456:2000 Plain and reinforced concrete - code of practice and we can also refer a book written by S.N Sinha i.e Reinforced Concrete design and let us come to conclude this lecture, Thank you!