Design of Structures

Lecture 12

Welcome to the UGC lecture series on B.Architecture. Here, we are having a look on the subject, Design of structure II. In this we are going to analyze a singly reinforced rectangular beam using the assumptions that have been made in the limit state method i.e the assumptions we have seen in the previous lecture. In this lecture, we are going to use the assumption for analyzing singly reinforced rectangular section. For the analysis, normally we all know that analysis means to find out the moment of resistance of the beam. So, normally when the beam is subjected to any kind of force, there is a moment induced in the structure. That moment which is used to resist the moment which is developed due to the external load, that is called as external moment. The moment which is induced in the structure, is called as internal moment. That is the moment which is used to resist the external moment. That moment is called as moment of resistance. When these two moments are equal, when both the moments are equal, the beam will be safe and also it will resist all the loads which are acting on the structure.

Now, we need to find out how to find out the moment of resistance of the beam. So here, as in the case of working stress method the moment is normally equal to the force x the distance. So, the moment of force is formed by the total compressive force offered by the whole concrete area above the neutral axis and the moment of force offered by the tensile force is due to the tensile reinforcement i.e Tu. So in order to find out the movement before the moment of resistance, here we are also having the three types of sections; one is the balanced section,another one is under reinforced section and the third one is over reinforced section.

Here we are having the Balanced sections, Under reinforced sections and Over reinforced sections. What do you mean by Balanced Sections? That we have already seen in detail in the working stress method of design of structures. Balanced section i.e concrete is normally designed on the basic assumption that both the materials stressed to their maximum permissible limit at the same time. Both the materials are subjected to their fullest allowable stress one and the same time, that section is called as Balanced section. In this section, the axis passes through, that section is called as a Critical neutral axis. That is called as Xu_{lim} i.e Xu_{max} or Xu_{lim}, that is maximum depth of neutral axis. In the case of unbalanced section, these are called unbalanced sections, in the case of under reinforced section i.e if less steel is used than required in the balanced section, then it is called under reinforced section. In this section, the steel is stressed to its maximum permissible value first while the concrete is stressed below its maximum permissible stress. This section will fail initially due to overstress in steel. So here, in the case of under reinforced section, the type of failure which occurs here is tension failure or ductile failure. In the case of an over reinforced section, if more steel than what is required for the balanced section is used, that is called as Over reinforced section, in such conditions, the section which starts failing initially due to over stress in concrete. So, the type of failure that occurs here is Brittle failure.

The depth of Neutral axis for the unbalanced section is normally taken by X_u. 'u' indicates Ultimate. So here, in the case of Limit state method, all the materials are designed to their limit state condition i.e ultimate loading condition. That's why they put X_u. Here it is X_u maximum i.e the Critical depth of neutral axis. Here it is called X_u Maximum depth of neutral axis of Xu_{lim} - limiting value i.e the maximum value of neutral axis. So here, not like in the case of limit state method but in the case of working stress method they allow these three sections for their design. But in the case of a limit state method, one of the difference between Limit state method and the working stress method is, in the case of limit state method they stopped avoiding designing Over reinforced section. Since the type of failure, it is a brutal failure, it's a sudden failure. That's why they are avoiding in designing over reinforced section. But in the case of under reinforced section, it shows the failure. We can easily rehabilitate that section. Its a start failure because the failure of section is a ductile failure, it is a tension failure. It shows the failure so that we can easily rehabilitate it. That's why they allowed to design the under reinforced section as well as the balanced section. So here, the over reinforced section has been avoided.

So, we are going to have only balanced section and under reinforced section. If an over reinforced section comes, what will happen? When over reinforced comes, the beams have to be redesigned i.e the beam has to be redesigned that as doubly reinforced section. In the case of Over reinforced section, we are using more amount of steel, automatically the concrete area will reduce. So in order to supplement concrete in resisting compressive stress, we need to provide the reinforcement at the compression zone. So that over reinforced section has to be designed as doubly reinforced section by providing the reinforcement at the compression zone. So, this is Asc and this one Ast. So, we can easily understand what is the reason for avoiding this over reinforced section. The over reinforced section, the nature of failure here it is duct time i.e brutal failure, it's a sudden failure, it fails suddenly. That's why they avoid designing an over reinforced section. If the 'Over reinforced section' comes in the case of design, it has to be designed as a doubly reinforced section. In the case of Over reinforced section, we are using more amount of steel. So, the concrete area automatically reduces, so there is no sufficient concrete to supplement the concrete in resisting the compressive stress. In order to resist that compressive stress in the concrete area, we need to place the reinforcement at the compression zone. It has to be redesigned as 'doubly reinforced section'. So this is the case of the 'Limit state method' they are avoiding Over reinforced section. How to find out Cu, Tu and the depth of neutral axis, the limiting value of the neutral axis is Xu_{max} or Xu_{lim}. This is the Maximum value of the neutral axis. Once you find out the maximum or the limiting value of the neutral axis, we can compare it with this X_u i.e the actual depth of neutral axis, we can easily conclude whether our section is balanced section or under reinforced section or over reinforced section.

How to analyze? Here, let us start with the rectangular section. First one; Singly reinforced rectangular section. Singly reinforced section means the section which is provided with the reinforcement and the tension zone only. This is the effective depth of the section and this one is b without the section and here this one is, if you take this to be the neutral axis for this section. This is the neutral axis and this is the strain diagram. The maximum strain at the topmost extreme fibre is 0.0035 as per IS 456:2000, the stress-strain relationship of concrete and the maximum strain at the tensile steel is;

fy ÷ 1.15Es + 0.002 = Et
Ec = 0.0035

In the case of stress diagram, this is a stress diagram. The stress is uniform upto a strain of 0.002 and thereafter the stress is varying and the maximum stress at the topmost extreme fibre is 0.446 fck and there is a compressive force Cu which is acting as the Cg at the compressive stress block. The Cu is the total compressive force offered by the whole concrete area above the neutral axis which is acting at the Cg of this compressive stress block, we are going to find out and Tu be the total tensile force which is offered by the reinforcement and the distance between these two forces is called Lever arm i.e Z, now we are going to find out. We first need to find out what is the total compressive force, what is the;

total compressive force offered DIVIDED the whole concrete area Cu

Normally force is; Stress x Area. Now, how to find out how to calculate the total compressive force offered by the whole concrete area above the neutral axis. In other words, Stress x Area can be Area of compressive stress block x b i.e the width of the section.

What is the area of compressive stress block? Here the area of compressive stress block consists of two different portions; one is the rectangular portion, another one is the parabolic portion. First we need to find out what is the depth of the rectangular portion and we can find out what is the depth of the parabolic portion, we can easily find out what is going to be the area. So here, in order to find out this one, we need to find out from the strain diagram. So here, at a distance Xu from the neutral axis the maximum strain is 0.005. At a distance, Xu the strain is Ec i.e at a distance A from the neutral axis, the strain is 0.002, I am taking this as A.

Xu ÷Ec = a ÷0.002

At the depth of Xu from the neutral axis, the strain is 0.0035 i.e Eu i.e

$Xu \div 0.0035 = a \div 0.002$

Over aim is to find out what is a. a is equal to

$Xu \div 0.0035 \times 0.002 = 0.57Xu$

The remaining is 0.43Xu. Now, we can easily find out, here there are two blocks. One is the rectangular block. The area of the rectangular block is; 0.446 fck x 0.43Xu. This is the area of rectangular portion + area of parabolic portion is $\frac{2}{3}$rd of the area of rectangle and considering this is an equivalent rectangle $\frac{2}{3}$rd of 0.57Xu x 0.446 fck. This is a rectangle, the horizontal portion is 0.446 fck, the vertical portion is 0.57 fck.

$(0.446 \text{ fck} \times 0.43Xu) + \frac{2}{3} \times 0.57Xu \times 0.446 \text{ fck}$

After multiplying this, [0.36fck Xu]b

$= 0.36\text{fck } X_u b \div$

We know that the force Cu = Stress x Area. What is stress here? 0.36 fck is the stress. After introducing the positive safety factor and after introducing the factor i.e 0.67 fck. So here, the area here is Xu and this will be b. The area of portion is $X_u b$ and the stress area is 0.36fck. Now, we need to find out;

To find Tu, what is Tu? Tu is the total tensile force offered by reinforcement. So which is equal to = stress x area. So here, the area is Ast (Area of tensile reinforcement). Stress is normally designed to fy. In the case of steel we are asked to introduce the factor of safety as 1.15, that has to be divided by 1.15 that is equal to 0.87fy Ast. So Tu = 0.85 Ast, Cu = 0.36 fck $X_u b$. So we know, these two forces.

Tu = Total Tensile force
= Stress x Area
= fy ÷ 1.15 x Ast

= 0.87fy Ast

Now, we can easily find out what is the depth of this neutral axis. The depth of neutral axis is; Xu. How to find out the depth of neutral axis? At the neutral axis, we all know that there is neither compression nor tension or it maybe total compressive force must be equal to Total tensile force. This is 0.36 fck X_ub, that we have found is equal to 0.87 fy Ast, that we have also found.

0.36 fck X_ub = 0.87 fu Ast

So here, Xu = 0.87fy Ast ÷0.36 fck b

This is the expression which is used to find out the depth of neutral axis. This is the actual depth of neutral axis. We can also find out the maximum depth of neutral axis. When you find out the maximum depth of neutral axis, we can compare the maximum neutral axis with the actual neutral axis, we can come to conclude whether our section is a balanced section or an unbalanced section. So, to find out Limiting value or Maximum Values of Xu i.e XuMax or Xu_{lim}. This Xu_{lim} which varies according to the steel reinforcement used in the design. There are three kinds of steel, one is Mild steel, another one is the Cold worked deformed bars i.e highly strengthened by Fe415 and Fe500. Now, to find out the maximum value of this one. So, here from the strain diagram. At a depth of Xu, the maximum stress is 0.0035. At a depth of d - Xu, the strain is;

fy ÷ 1.15 + 0.002

We need to find out the maximum strain. We can easily find out what is the limiting value of the depth of neutral axis from this expression i.e

Xu ÷Є Cu = d -Xu ÷Є Tu

Take this as Є Cu and this as ЄTu. But we need to first find out what is Є Cu equal to 0.0035.

Є Cu = 0.0035
Є Tu = fy ÷1.15Es + 0.002

This is for Mild Steel. Mild. fy for mild steel is 250 ÷ 1.15 x 2 $(10)^5$ + 0.002. The strain is 0.00308

$= 250 \div 1.15 \times 2(10)^5 + 0.002 = 0.00308$

$Xu_{lim} \div 0.0035 = d - Xu_{lim} \div 0.00308$

$Xu_{lim} = 0.531\ d$

This is for Mild Steel and for torsteel i.e for Fe415, it is the way of finding. This is the way of finding the next one, We need to find out Xu_{lim} for Fe415 i.e

$Xu_{lim} \div \epsilon\ Cu = d - Xu_{lim} \div \epsilon Cu$

$\epsilon\ Cu = 0.0035$

$\epsilon\ Tu = fy \div 1.15Es + 0.002$

fy for Fe415 is $= 415 \div 1.15 \times 2\ (10)^5 + 0.002$

$= 0.00380$

$Xu_{lim} \div 0.0035 = d - Xu_{lim} \div 0.00380$

So Xu_{lim} for Fe415 is;

$Xu_{lim} = 0.479d$

Then for fe500. So $Xu_{lim} \div \epsilon\ Cu = d - Xu_{lim} \div \epsilon\ Tu$. So here,

$\epsilon\ Cu = 0.0035$

$\epsilon\ Tu = fy \div 1.15Es + 0.002$

= 500 ÷1.15 x 2(10) power 5 + 0.002

$= 0.00417$

$Xu_{lim} \div 0.0035 = d - Xu_{lim} \div 0.00417$

So, Xu_{lim} for given grade of steel, here it is, 0.456d

So this is the way of calculating the Xu_{lim}. So here, we have calculated Cu. How to calculate the value Cu i.e 0.36fck X_ub i.e from the stress diagram, total compressive force Cu = area of the compressive stress block x b. Here, the area of compressive block, here the stress block consists of different portions; one is rectangular portion, another one is parabolic portion. The height of the parabolic portion has been found to be 0.57Xu and the remaining one is 0.43 Xu and the area of rectangular portion and the area of parabolic portion, that has to be multiplied by width of the section. We have got 0.36fck X_ub where 0.36 is the stress and the area is X_ub and total compressive force is stress x Area. Normally the material is stressed to its maximum ultimate or yield stress fy x Ast.

In the case of limit state method, we are introducing a factor of safety as 1.15 i.e 0.87fy Ast. This is the way of calculating Cu and Tu. Then to find the neutral axis depth and the neutral axis, Cu must be equal to Tu i.e at the neutral axis the Cu must be equal to Tu. 0.36fck X_ub is equal to 0.87fy Ast.

Xu = 0.87fy Ast ÷0.46fck b

So this is the expression for finding out the actual depth of neutral axis. So once, if you know the actual depth of neutral axis, we need to know what is the maximum depth of neutral axis, that is called as the limiting depth of neutral axis. Once we find out what Xu is, we can compare it with Xu, we can find out if our section is under reinforced or over reinforced or the balanced section.

So here, in order to find out the maximum depth of neutral axis, we need to use the strain diagram. At a depth of X_u, the maximum depth, the maximum strain is 0.0035. At a depth of d - X_u from the neutral axis, the maximum strain is ϵT_u.

ϵ Tu = fy ÷1.15Es + 0.002

So this limiting or maximum depth of neutral axis can be found according to the steel reinforcement used in the design. There are three types of steel. One is Mild steel, another one is Fe415, third one is Fe500. Here in this way, we have calculated Xu_{lim} for the Mild steel and we have followed the same steps and found the Xu_{lim} for Fe415 i.e 0.479d. In the same way we have found the Xu_{lim} for Fe500 i.e 0.456.

I have tabulated these values for different types of steel. The fy is 250, Fe 415 415, for Fe500 it is 500. Xu_{lim} I have tabulated, for the corresponding maximum strain. For the corresponding maximum strains, we have found the Xu_{lim}, then we compare this Xu with Xu_{lim}. We can conclude whether our section is balanced section, under reinforced section or over reinforced section.

Let us summarize this lecture. In this case we have seen so far; Analyzing the rectangular section i.e the Singly reinforced rectangular section. We have found the total compressive force offered by the whole concrete area above the neutral axis. We have seen the total tensile force offered by the tensile reinforcement and we have seen how to calculate the depth of neutral axis by equating the total compressive force x the total tensile force and we have calculated the maximum value of depth of neutral axis i.e Xu_{lim}. The Xu_{lim} which is based on/ which is according to the type of reinforcement used in the design. There are three reinforcements normally we are using; one is Mild steel, another one is Fe415, another version is Fe500. This is as per IS 456, Xu_{lim} for Mild steel 0.531d, Xu_{lim} for Fe 415 is 0.479d, Xu_{lim} for Fe500 is 0.456d. This is the thing we have found in this lecture.

Then, the questions pertaining to this lecture are; Determine total compressive and tensile force for singly reinforced rectangular section, Find the limiting values of Xu for the various grades of steel Xu_{lim}. Once you know how to find out this Xu_{lim}, we can easily find out whether our section is balanced or unbalanced section.

References, we need to refer; IS 456:2000 Plain and reinforced concrete - code of practice and we can also refer a book written by S.N Sinha i.e Reinforced Concrete design and let us come to conclude this lecture, Thank you!