Design of Structures II

Lecture 10

Welcome to UGC lecture series in B.Architecture. Here we are having a look on Design of structures II. In the previous lecture we have seen the analysis of doubly reinforced flanged section when the neutral axis lies within the flange and also in addition to that we have also seen some problems on singly reinforced flanged section when the neutral axis lies within the flange and when the neutral axis lies outside the flange and thereafter we have seen the analysis of doubly reinforced section when the neutral axis lies within the flange, we have analyzed the section and we have found the critical depth of neutral axis and actual depth of neutral axis and we have compared it to the depth of the section with the actual depth of neutral axis. Here the x is less than Df, we have found the moment of resistance of the corresponding section. Now, in this lecture, we are going to have an analysis of doubly reinforced flanged beam, when the neutral axis lies outside the flange and also, we are going to find out what is the moment of resistance. In addition to that, we are going to work some examples on doubly reinforced flanged beam and on neutral axis within the flange as well as outside the flange. So here, this is case II,

Analysis of Doubly Reinforced T and L Section – 1

How to find out this expression to find out the x of the flanged beam and how to find out the moment of resistance of the beam.

Now, x >Df, neutral axis lies outside the flange. So here, this is the beam. A T beam which is doubly reinforced and this reinforcement is called as Area of compression reinforcement which is placed at d' from the topmost extreme fibre and this is the area of tension reinforcement which is placed at d from the topmost extreme fibre and bw is the width of the flange and Df is the depth of the flange. Now, you take this as the critical neutral axis and I am taking this as the equivalent stress diagram of balanced section. In the case of balanced section, normally here xc is the critical depth of neutral axis and (d-xc) is the remaining depth and in the case of balanced section, both the materials are stressed to their maximum permissible value one and the same time.

To find out xc, we are using the similar triangle property at a depth of xc, it is σ cbc and at a depth of (d - xc), the stress is σ st/m.

This is the formula used to find out the critical depth of neutral axis. Now here, the neutral axis lies outside the flange. I am taking this as the equivalent stress diagram of unbalanced section. I am considering this as x and this by d - x. So here, when the neutral axis lies outside the flange, I have already told you that, it has to be designed as a T section, it has to be analyzed as a T section. So here, I am dividing the portion of compression zone into 2 different portions; one is the flanged portion and the other is remaining depth portion. So this one is Df and this one is (x- Df). So here, in the case of unbalanced section, to find out x. The neutral axis is situated at the centre of gravity of the section. For finding out, I am going to, to find out the actual depth of neutral axis, I'm equating the moment of area on compression side to the moment of equivalent concrete area on Tension side. So this is the transformation of area of steel into the equivalent concrete area of the steel. So, that's why I have considered concrete area below the neutral axis surrounding the tensile steel. The moment of area of compression steel, normally moment of area is;

Area x C.G distance.

What is the area? So here, I have divided this portion into two portions; first one is the flanged portion i.e;

area of flanged portion = bf.Df

This is the moment of area of the flanged portion into distance of C.G of this flanged portion, here it is;

$\frac{Df}{2}$, it occurs at the centre.

The distance of C.G of the flanged portion from the neutral axis is;

(x - Df/2)

So, area of flanged portion = bf.Df(x - Df/2)

This is the moment of area of the flanged portion plus the moment of area of the web portion is;

bw(x - Df)

Area of the web portion is; bw (x - Df) and the distance of C.G of this flanged portion is, ($\frac{x-Df}{2}$) Since the steel is placed at the compression zone, I need to deduct the moment of area of the compression steel minus Asc (x - d'). So, Asc is the area of the compression reinforcement which is placed at d' at the topmost extreme fibre and the distance of C.G of this compressive reinforcement is (x - d'). I have subtracted the moment of area due to the compression

reinforcement and I am also going to add an equivalent concrete area of the compression reinforcement, mAsc (x - d').
The compression in bars vs the compressive resistance in concrete is taken into account, shall be the calculated stress multiplied by 1.5 times the modular ratio.

$\sigma sc = 1.5m\ \sigma cbc$

Since the stress is multiplied by the 1.5 times the modular ratio, here also it has to multiplied by 1.5m. Which is equal to, the moment of the whole concrete area i.e moment of area of the flanged portion.Moment of area of the compression portion. This is the moment of whole concrete above the neutral axis. This is the moment of area due to the compression reinforcement. This is the moment of equivalent concrete area of the compression reinforcement, which is equal to the moment of equivalent concrete area on the tension side, (d - x).

mAst is the equivalent concrete area of the tensile steel and a distance of C.G of the tensile steel from the neutral axis, (d-x). In this expression, since the moment of area of web portion is too small as compared to flanged portion. The moment of area of the web portion is very small as compared to the flanged portion. Hence, the moment of area of the web portion has to be neglected. So, after neglecting this one, it can be further rewritten as;

$bf .Df (x - Df/2) + (1.5m - 1) Asc (x - d') = mAst(d - x)$

This is the equation which is used to find the value of x. Now you compare x with Df. If x >Df , it lies i.e neutral axis lies within the flange and also we need to compare x with the xc. If x <xc, it is under reinforced section. If x >xc, it is over reinforced section. Now, the moment of resistance of the section which is compressive force C_1 into Z_1 plus C_2 Z_2.

$Mr = C_1 Z_1 + C_2 Z_2$

So here, we have neglected the compression portion. We have the resultant stress diagram as the only flange portion. I am taking σ cbc as the stress at the topmost extreme fibre and σ cbc'' is the stress or the σ cbc' is the stress at the bottom of the flange and this is the reinforcement which is placed at T' at the topmost extreme fibre. At the place, it is taken as σ'', this is taken as σ' and this is taken as σ cbc'. Now, we need to find out the moment of resistance. So, C_1 be the total compressive force offered by the whole concrete above the neutral axis i.e at the flanged portion which is acting at Df/2 from the topmost extreme fibre and the corresponding tensile force is T_1. The lever arm is (d - Df/2). Since it is not correct. So here, it is acting at $\bar{y}$, since it is a trapezium, we have already discussed in the previous one. This is σcbc, this one is σ cbc' and this one is Df and $\bar{y}$ here it is,

The corresponding lever arm is (d-$\bar{y}$). Another force is, since the steel is placed at the compression zone. C_2 be another force due to the compression reinforcement which is acting at d', the topmost extreme fibre and T2 be another force which is used to compensate C_2 and the distance between these two resultant forces (d - d') is taken as Z_2, this is taken as Z_1. Now, we need to find out Lever arm i.e compressive. Here,

$Mr = C_1 \times Z_1 + C_2 \times Z_2$

bfx is the area of the compression portion. Area x stress is the force, into the lever arm is (d - $\bar{y}$);

This is the moment of resistance , (or)

$AsT_1\ \sigma st(d - \bar{y}) + Ast2\ \sigma st (d - d')$

This is another expression. Now, we can use this as the only expression to find out the moment of resistance of both under reinforced and over reinforced section. If it is under reinforced section, steel is stressed to its maximum permissible limit. We know the σ st since σ st the permissible value of σ st but σ cbc is not a permissible limit, it is stressed below the permissible stress, it is always less than the permissible stress. So, we can substitute here, we can find out the moment of resistance of the under reinforced section. So, this is the analysis of doubly reinforced flanged beam where the neutral axis lies outside the flange. This is the moment of resistance.

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Now, we need to solve a problem using this expression. Now here, the moment of resistance of the doubly reinforced section. Now, we have to do some problems;

So here, the beam, it's a T Beam. Find the moment of resistance of T Beam of effective depth of 600. So this is taken as, Area of tensile steel and d = 600mm and flanged with this 1500 mm, bf = 1500mm. Depth of flange is 150mm. Tensile steel is 1964, Ast = 1964 mm^2 at the bottom and compression steel 1140. So here, the compression steel is placed at the top at a distance d', it’s given in the problem and the compression reinforcement Asc = 1140 mm^2 and here the steel is placed at the top i.e 50mm from the top, d' = 50mm. σ cbc = 7 N/mm^2 and σ st = 230 N/mm^2. What is the moment of resistance of the beam?

Now, first case, first to find out xc of the section;

After substituting all the values, we get;

$x_c = 173.16$ mm

To find x : We first assume the neutral axis lies within the flange. Assume x is less than or equal to Df. We are not aware of whether the neutral axis lies within the flange or outside the flange. First I shall begin from scratch i.e $x \leq D_f$

After substituting all the values, we get;

x = 119.5mm

Now, we need to compare x with Df. x is less than Df. Also, $x < x_c$. So, neutral axis lies within the flange. So our assumption is correct. Here, $x < D_f$, it is under reinforced section. So here, since it is an under reinforced section, σ_{st} is known since the steel is stressed to its maximum permissible value while the concrete is stressed below its permissible value. We need to find out, To find actual σ_{cbc};
From the depth of neutral axis we need to find out.

$\sigma_{cbc}(ac) = 4.29 \text{ N/mm}^2 < \sigma_{cbc}'$

Now, we need to find out;

To find σ_{cbc}' at a depth of Df from the top.

This is σ_{cbc}'. Let's say σ_{cbc}' is 0.56 N/mm^2

The fifth step, to find σ_{cbc}'' at a depth of d' from neutral axis i.e

$= 3.24 \text{ N/mm}^2$

Now, we find out the moment of resistance. To find Mr of the section;

After substituting;

= 201.314 kNm

So here, x = 173.31, 119.5 mm. $\sigma_{cbc} = 4.29$, $\sigma_{cbc}' = 2.495 \text{ N/mm}^2$. Here x is less than Df, not greater than Df. Since x is less than Df, lies within the flange, actual $\sigma_{cbc} = 4.295$ and

$x < D_f$

$= 245.09$ kNm

This is the way we find the moment of resistance of the beam. This is the first case we have seen and the second case with the second problem we can do it when x is greater than Df.

Let us summarize this lecture, in this lecture we have seen the analysis of doubly reinforced flanged beam by working stress design method. In this case we have seen two cases; x is less than or equal to Df and x is greater than Df and we have also derived the respective expressions for the moment of resistance and also we have worked out some examples for finding out the moment of resistance of the beam. But we have worked out only one problem in the case of moment of resistance of the beam i.e case 1 i.ex is greater. I am going to give you another problem that you could practice at home, to find out the moment of resistance of the section. The questions for this lecture - Determine the expression for depth of neutral axis for the doubly reinforced flanged beam when N.A lies outside the flange and also find the corresponding moment of resistance. So here, we can also do another problem, you can try solving this in the case where the neutral axis lies outside the flange and find out the moment of resistance of the beam. Please try to solve this problem, in turn practice.

In this lecture, the references we can refer IS456:2000 Plain and reinforced concrete - Code of practice and we can also refer a book written by S.N Sinhai.e Reinforced concrete design. With this we come to the end of this lecture. Thank you!