FAQ's

1. Explain the concept of double integration?

The double integration method is a powerful tool in solving deflection and slope of a beam at any point because we will be able to get the equation of the elastic curve.

In calculus, the radius of curvature of a curve y = f(x) is given by

$$\rho = \frac{\left[1 + (dy/dx)^2\right]^{3/2}}{\left|d^2y/dx^2\right|}$$

In the derivation of flexure formula, the radius of curvature of a beam is given as

$$\rho = \frac{EI}{M}$$

Deflection of beams is so small, such that the slope of the elastic curve dy/dx is very small, and squaring this expression the value becomes practically negligible, hence

$$\rho = \frac{1}{d^2y/dx^2} = \frac{1}{y^{n}}$$

Thus, EI / M = 1/y''

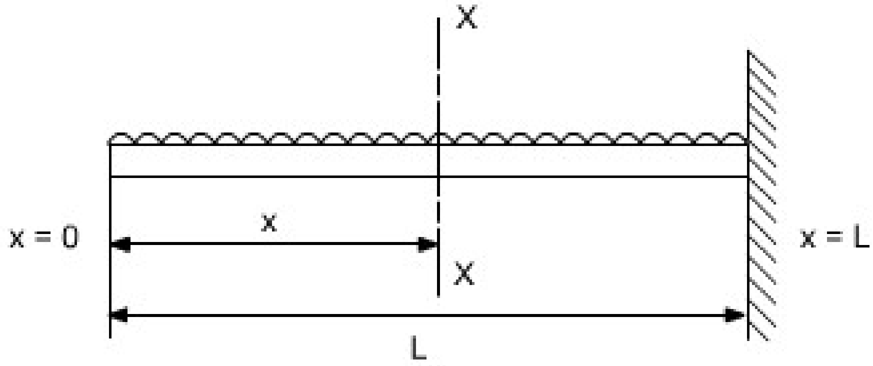
$$y'' = \frac{M}{EI} = \frac{1}{EI}M$$

If EI is constant, the equation may be written as: EIy''=M

where x and y are the coordinates shown in the figure of the elastic curve of the beam under load, y is the deflection of the beam at any distance x. E is the modulus of elasticity of the beam, I represent the moment of inertia about the neutral axis, and M represents the bending moment at a distance x from the end of the beam. The product EI is called the flexural rigidity of the beam.

The first integration y' yields the slope of the elastic curve and the second integration y gives the deflection of the beam at any distance x. The resulting solution must contain two constants of integration since EI y" = M is of second order. These two constants must be evaluated from known conditions concerning the slope deflection at certain points of the beam. For instance, in the case of a simply supported beam with rigid supports, at x = 0 and x = L, the deflection y = 0, and in locating the point of maximum deflection, we simply set the slope of the elastic curve y' to zero.

2. Find the slope and deflection of cantilever beam subjected to uniformly distributed load?



$EI(d^2y/dx^2)=-M_{xx}$

$$M_{xx}=-(wx)(x/2)$$

$$=-wx^2/2$$

$$\text{Equation 1}=EI(d^2y/dx^2)=(wx^2/2)$$

Integrating (1) w.r.t x,

$EI(dy/dx)=(wx^3/6)+c_1 \rightarrow$ Equation 2

Integrating (2) w.r.t x,

$EI.y=(wx^4/24)+c_1x+c_2 \rightarrow$ Equation 3

The boundary conditions are

When $x=l, dy/dx=0 \rightarrow$ (i)

When $x=l, y=0 \rightarrow$ (ii)

Using boundary condition (i) in equation 2,

$C_1=-(wl^3/6)$

Using boundary condition(ii) in equation 3,

$C_2=(wl^4/6)-(wl^4/24)=wl^4/8$

Hence general equation for slope is

$EI(dy/dx)=(wx^3/6)-(wl^3/6) \rightarrow A$

The general equation for deflection is

$EI.y=(wx^4/24)-(wl^3/6)x+(wl^4/8) \rightarrow B$

To get slope at B, put x=0 in A

$EI(dy/dx)=-(wl^3/6)$

$(dy/dx)=-(wl^3/6EI)$

$\theta_B=-(wl^3/6EI)$

To get deflection at B, put x=0 in B

$EI.y=(wl^4/8)$

$y_B=(wl^4/8EI)$

3. Find the slope and deflection in cantilever beam of span 5m subjected to a point load of 10kN at the free end? Assume EI as constant.

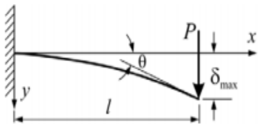
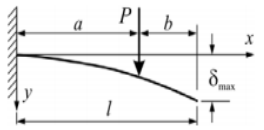
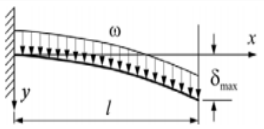
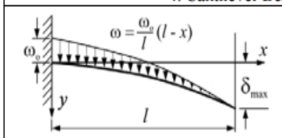
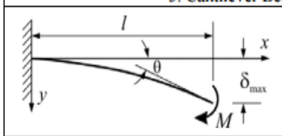
l = 5m; P=10kN

$\theta = Pl^2/2EI = (10x5^2)/2EI = 125/EI$

$y = Pl^3/3EI = (10x5^3/3EI) = 416.67/EI$

4. Give the expression for slope and deflection of cantilever beam subjected to various load conditions?

BEAM DEFLECTION FORMULAE

BEAM TYPE	SLOPE AT FREE END	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM DEFLECTION
1. Cantilever Beam – Concentrated load P at the free end			
	$\theta = \frac{Pl^2}{2EI}$	$y = \frac{Px^2}{6EI}(3l - x)$	$\delta_{max} = \frac{Pl^3}{3EI}$
2. Cantilever Beam – Concentrated load P at any point			
	$\theta = \frac{Pa^2}{2EI}$	$y = \frac{Px^2}{6EI}(3a - x)$ for $0 < x < a$ $y = \frac{Pa^2}{6EI}(3x - a)$ for $a < x < l$	$\delta_{max} = \frac{Pa^2}{6EI}(3l - a)$
3. Cantilever Beam – Uniformly distributed load ω (N/m)			
	$\theta = \frac{\omega l^3}{6EI}$	$y = \frac{\omega x^2}{24EI}(x^2 + 6l^2 - 4lx)$	$\delta_{max} = \frac{\omega l^4}{8EI}$
4. Cantilever Beam – Uniformly varying load: Maximum intensity ω_o (N/m)			
	$\theta = \frac{\omega_o l^3}{24EI}$	$y = \frac{\omega_o x^2}{120lEI}(10l^3 - 10l^2x + 5lx^2 - x^3)$	$\delta_{max} = \frac{\omega_o l^4}{30EI}$
5. Cantilever Beam – Couple moment M at the free end			
	$\theta = \frac{Ml}{EI}$	$y = \frac{Mx^2}{2EI}$	$\delta_{max} = \frac{Ml^2}{2EI}$